Journal
of
The Franklin Institute
Devoted to Science and the Mechanic Arts

Vol. 253 APRIL, 1952 No. 4

RADIO'S MEMORABLE ANNIVERSARY *

BY

THOMAS COULSON[1]

From the dawn of history the two principle agencies for the dissemination of knowledge have been books and communication methods. Once writing was established, its methods of preservation underwent continuous improvement until the invention of printing made the multiplication of books a relatively simple and popular means for the diffusion of knowledge. Yet, other means of communication remained stationary for centuries. The methods available to the founders of the American Republic were substantially the same as those used in Biblical times.

The Nineteenth Century was to witness greater technological advances than any previous period of similar length, and it is not surprising to learn that existing methods of communication were improved and new ones introduced. First, came the semaphore for visual signalling, which reduced the time required for transmitting messages over short distances; then came the electric telegraph to extend the area which could be covered; and finally, came the telephone for the transmission of speech over longer distances. At the close of this century man was enjoying communication facilities undreamed of by the Founding Fathers. There seemed nothing further to discover or invent which would lead to the quickening of transmission or its diffusion over a wider area.

As the century drew toward its close a few scientists were experimenting with the electrical waves which Clerk Maxwell had said could travel through the air with the same speed as light. Maxwell did not produce any such waves, but he said they were there and one day an

* From the talk presented on the General Electric Science Forum Program, over WGY, Schenectady, December 12, 1951, honoring the 50th anniversary of trans-Atlantic radio.

[1] Director of Museum Research, The Franklin Institute, Philadelphia, Pa.

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.)

experimenter would find them. Heinrich Hertz found them twenty-three years later in his laboratory at Bonn, and his discovery inspired a number of other scientists to pursue the study. Although there were several improvements in the crude apparatus devised by Hertz there was no indication in the results obtained that would induce anyone to proclaim that the world had arrived at the eve of a new era in communication.

The trans-Atlantic cable by which means of rapid communication was conducted between the New World and the Old had been regarded as an engineering triumph—and rightly so. It represented a vast organization of many skills and much capital, and it reflected the infinite patience of man in overcoming technical and natural obstacles. The cable was such a noteworthy advance upon any preceding trans-oceanic method of communication that users were willing enough to overlook any of its deficiencies. Even when this continent and Europe had been connected by cable, messages were frequently delayed because of the limited capacity of the cable to carry them; and, when the distant end of the cable had been reached, the messages had to be relayed over thousands of miles of wire stretching in all directions. To convey an important news item to the capitals of Europe, implied the dispatch of as many telegrams as there were capitals. Many places remained inaccessible to the receipt of rapid delivery because no wire reached them. Nor could anything moving, as a ship, be reached. Travellers by sea had to reconcile themselves to isolation from the rest of the world until they arrived at some port.

Although the sparks produced by Hertz oscillators were to be heard crackling in many laboratories in Europe and America, the resonators for receiving them at a distance were not sufficiently sensitive to inspire any grand hopes of achieving widespread broadcasts over long distances in the immediate future. The possibility that these sparks might emerge into the nucleus of an entire new conception in the realm of instruction and entertainment had not occurred to the most optimistic investigator. In the '80's even the scientific world had been so skeptical about the idea of electromagnetic waves traversing space, that David Edward Hughes, who had demonstrated a microphone before the Royal Society in London, was so discouraged at being unable to convince his fellow scientists of the truths of these waves, that he refused to read a paper on the subject. Twenty years later, the waves were not only accepted as practical, but they were being transmitted and received in many laboratories over distances measured in feet. In February, 1896, Guglielmo Marconi appeared to be in advance of the movement when he took out the first British patent on wireless telegraphy. Four years later, he had succeeded in transmitting signals from St. Catherine's Point, on the Isle of Wight, to the Lizard in Cornwall, a distance of 198 miles. This represented rapid progress, and the

success inspired him with the hope that he might be able to transmit and receive messages across the Atlantic Ocean.

This conception was so much bolder even than the trans-Atlantic cable that it bordered on the fantastic. Instead of proposing to have a single channel over which messages could be passed for distribution over a vast network of wires, it was proposed to disseminate messages instantly in all directions over the entire world.

Such an enormous expansion of communications as this promised to be was a grotesque proposal to offer to a world educated to the marval of the restricted trans-oceanic cable. It was such a daring enterprise, charged with so many problems and hazards and intangibles that the young inventor decided to forego the fanfares, flagwaving, and banquets that had marked the opening of the Atlantic cable. He was by no means sure of success, but he was inspired by a faith, a boundless faith, that patient and painstaking investigation would reveal many unsuspected secrets in these mysterious electric waves. He was already becoming accustomed to seeing public inertia break down before the demonstrations of their practical applications.

Some of the experts of the Signal Corps as well as the general public had scoffed at the contention that these electromagnetic waves would pass through buildings and mountains, but demonstrations had convinced them of their error. Sailors, more than anyone else, wanted a method of communication which would enable them to keep in touch with the shore, but invisible waves did not appear to be the solution to their problem. How could they penetrate a fog? Or fail to lose direction in darkness? Being released indiscriminately into the air, the waves would be distorted and dispersed by storms. Demonstrations had proved these fears to be false, and most of the wireless experiments were conducted between ship and shore, or between ships in motion. But formidable obstacles still loomed before wireless telegraphy over great distances came within the bounds of probability.

Marconi had built a transmitting station at Poldhu, in Cornwall, England, and he contemplated erecting a receiving station somewhere on the North American continent. Regarding the location of these two points on a terrestrial globe showed clearly enough that the curvature of the earth would raise a gigantic wall of water between them. The factor around which most speculation centered was whether the electrical waves could penetrate this water barrier or whether they would follow the curvature of the earth. Neither seemed at all probable. Even if the water proved to be no barrier, the probability was that the waves would glide off into space.

However, encouraged by his small success in receiving signals over a distance of nearly 200 miles, Marconi resolved upon making the experiment of trying to receive them over ten times that distance. He had no theoretical knowledge to guide him, nor had anyone else, so

that he had no alternative but to set up an American station and make the test. Satisfied that the Poldhu station would serve for the transmitter, he proposed to build the receiving station at Cape Cod, Mass. At each station a ring of masts, 200 ft. high, supported the aerial wires, which were brought in to the apparatus in the center of the ring. In September, 1901, the Poldhu aerial was blown down by a gale. Two wooden masts, 170 ft. high, were hastily substituted, with a fan-shaped aerial hung between them. In November, the Cape Cod aerial was blown down.

When Marconi sailed for America with these disasters fresh in mind, he might very well have been pardoned if he had allowed his thoughts to linger pessimistically over the misfortunes which had attended the laying of the Atlantic cable. He may have recalled how the cable had broken several times, provoking months of delay. Then, after the cable had been successfully laid, the electric impulses had failed to make their way over it. The project was abandoned for seven years before it was resumed. Marconi could have anticipated that his own enterprise might encounter a similar series of unpredictable frustrations and delays before success was realized.

Yet, within six days of landing he had established a receiving station on the summit of Signal Hill, overlooking St. John's harbor, Newfoundland. He had abandoned his thought of having an elaborate ring of masts to support an aerial, and hoped to achieve parallel results with the aid of some balloons and kites he had brought for the purpose. The balloons were found to be unmanageable in the turbulent December air, but a kite was flown from which the receiving aerial, about 500 ft. long, was suspended. This aerial was secured to a mast and connected by a wire to the receiving apparatus. A self-restoring coherer (Marconi's own improvement upon Branly's original coherer) and a pair of telephones formed part of the apparatus. The latter were preferred to the usual arrangement of having the coherer signals recorded automatically on paper tape through a relay and Morse instrument. The human ear, being more sensitive than the recorder, would pick up faint signals to which the mechanical device might not respond.

Marconi had arranged with J. A. Fleming (later better known as Sir Ambrose), who was in charge of the Poldhu station, for signals to be transmitted at a certain speed and at certain hours on receipt of a message announcing that the receiving set was in readiness. On December 11, 1901, Marconi dispatched a cable giving instructions for the transmission to start at noon on the following day.

As the clock moved toward mid-day on Thursday, December 12, Marconi took his seat at the receiving apparatus with a telephone at his ear. At such a dramatic moment one would have expected an audience to have assembled, waiting with suppressed excitement for

the outcome of the experiment. In point of fact, Marconi and his two assistants were alone. The experimenter was a shy reserved man who shunned publicity unless he had some definite statement to make. He had no desire to suffer public discomfiture over the crucial experiment on which hinged new and revolutionary communications among all nations on earth. If his conception of trans-oceanic wireless should collapse like a bubble the failure could do irreparable harm to the infant science and to the commercial company he had recently formed to exploit it. Practical wisdom dictated silence in England, but the matter could not be held in secret in a small community like that of St. Johns, where the flying in midwinter of balloons and kites was not an every-day occurrence. However, no curious observer climbed the snow covered slope of Signal Hill that day.

At twelve o'clock Marconi began listening for his signal. For half an hour he listened and heard—nothing. He began groping through the air for the English signal. He had no calibrated dial such as we have today on our radio sets for locating a desired station. There would have been no advantage in having one, since he did not know the wave measurement of the Poldhu transmitter. There was no device by which the wave could be measured. So Marconi had to search through space for his signal.

He listened and listened. Not a sound could he detect. He inspected his instrument and found it was perfect. Had something gone wrong in Poldhu to interfere with transmission? He resumed listening but again without result. Doubts began to assail him. Was the curvature of the earth an insurmountable barrier? Had he not been too optimistic in supposing that an invisible wave could travel through the air across an ocean and find a single copper wire dangling from a kite nearly 2000 miles away? There appeared substantial reasons why the electromagnetic waves should never reach St. Johns, and only very slender hopes that they might.

He patiently resumed his blind tuning in search of the elusive signal. After another half an hour of searching he thought he detected a sound. Listening intently he imagined this sound was repeated, three short dots—the letter S in Morse code. It was the prearranged signal. Marconi was receiving the first wireless signal that had traversed the Atlantic.

How did he take it? Remember this young man (he was only twenty-seven years old) was the son of an Italian father and an Irish mother, a combination which gave promise of offspring with ebullient natures. No one would have been astonished had there been a violent commotion on the top of Signal Hill that afternoon. But all was calm and still. In recalling that thrilling moment some time later, Marconi said: "I could not be satisfied without corroboration. 'Can you hear anything, Kemp?' I said, handing the receiver to him. Kemp heard

the same thing I did, and I knew I had been absolutely right in my anticipation."

That was all. No elation, no exultation, no jubilant telegrams fired off in all directions to the press. So far as we know there was not even an extra glass of wine at dinner that night.

Next day, Marconi and Kemp took up their listening again. They could hear nothing but those three dots, repeated with monotonous regularity. But could any music have sounded sweeter? There was only one radio station in the world which could be sending out that signal. Distance had been annihilated, the electromagnetic waves were passing serenely around the earth, the skeptics would be confounded. All that was now required to make wireless telegraphy a practical commercial enterprise was the refinement and improvement of the sending and receiving apparatus.

The history-making experiment, headline stuff for the newspapers, was not immediately announced to the world. Marconi kept it secret until he learned what another day might bring forth. When the next day's experience provided confirmation of their success, he issued a restrained statement in which he characteristically went out of his way to pay a tribute to the men whose earlier discoveries had paved the way for his investigations. The news reached New York on a peaceful Sabbath morning. Instantly reporters were rushing helter-skelter northward to get the story from the young man who had made a vision become a reality. The gist of the reports they sent back was that the visionary had spent $200,000 in sending three dots across the Atlantic. Even at that price they were cheap.

That was fifty years ago. The old sparks no longer crackle in the transmitting rooms. Their place has been taken by softly glowing tubes that silently convey their messages. Science moved much too quickly for radio to remain long unchanged. Anything that will transmit light and sound through space must necessarily have a multiplicity of uses. Enlivened by constant progress, refreshed by invention, radio retains a youthful vigor, but as changes come and the substituted parts are laid aside, as we learn the new names which progress brings, one name must remain outstanding, the name of the young man with a faith and vision—Guglielmo Marconi.

A GENERAL THEORY OF LINEAR SIGNAL TRANSMISSION SYSTEMS

BY

L. A. ZADEH[1]

ABSTRACT

The analysis and synthesis of signal transmission systems are usually conducted either in the time domain or in the frequency domain. In this paper a more general approach based on the resolution of signals in terms of some suitable, but otherwise arbitrary, set of component signals is outlined. A signal transmission system is characterized by a so-called characteristic function, of which such commonly used descriptive functions as the impulsive response, unit step response, and the system function are special cases. The general input-output relations are interpreted in geometrical terms by using the function space approach. The concept of ideal filter is introduced via the idempotent property of projections in function space, and the significance of this concept is indicated. Stability, operational symbolism, product relation, and inverse systems are briefly discussed.

INTRODUCTION

The analysis and synthesis of signal transmission systems such as filters and modulators are generally conducted either in the time domain or in the frequency domain. In employing the time domain approach, the various time-dependent quantities describing the system are dealt with directly and use is made of such auxiliary concepts as the impulsive response (Green's function) or the unit-step response. On the other hand, in employing the frequency domain approach the time-dependent quantities are dealt with indirectly through their respective frequency spectra. This implies the use of the familiar techniques of the Laplace transformation, the Fourier transformation, and Heaviside's operational calculus.

In this paper we shall outline a more general theory which contains the above approaches as special cases. The theory is based essentially on the resolution of the input signal into a set of "elementary" or component signals of some suitable, but otherwise arbitrary, type. The general relations obtained on this basis are useful in two respects. First, they serve to unify the various standard techniques of analysis of signal transmission systems and contribute to one's understanding of the behavior of such systems. More important, these relations place in a new perspective some of the basic problems involved in the transmission and reception of signals, and thereby suggest novel approaches to some of these problems.

Many of the mathematical techniques used in the theory are commonly employed in quantum mechanics, though in a different form and for different purposes. The dissimilarities are due largely to the special

[1] Assistant Professor of Electrical Engineering, Columbia University, New York, N. Y.

character of signal transmission systems and the nature of the problems associated with the transmission and reception of signals.

In the present paper the theory is discussed only in general terms. More detailed discussions of various aspects of the theory and its practical applications will be given in subsequent papers (1, 2).[2]

BASIC THEORY

The object of our discussion is a non-specified linear signal transmission system N of the form shown in Fig. 1. The input and output

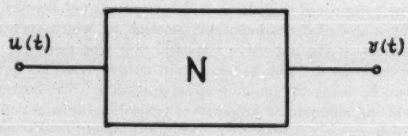


FIG. 1. Schematic representation of a signal transmission system.

signals are denoted by $u(t)$ and $v(t)$, respectively. It will be assumed throughout, except where otherwise indicated, that N is a variable system and that it is at rest (but not necessarily fixed) prior to the application of the input signal.

Before proceeding with the discussion, it will be helpful to formulate a precise definition of a linear system.

Definition.—A system N is called a linear system if it has the additive property, that is, if the response of N to an input of the form

$$u(t) = c_1u_1(t) + c_2u_2(t), \tag{1}$$

where c_1 and c_2 are arbitrary constants, is of the form

$$v(t) = c_1v_1(t) + c_2v_2(t), \tag{2}$$

where $v_1(t)$ and $v_2(t)$ are the responses to $u_1(t)$ and $u_2(t)$, respectively.

All methods of analysis of linear systems are based in one way or another on the additive property of such systems. In particular, all commonly used methods for finding the response of a system to a given input are essentially variants of the following basic procedure.

1. The input is resolved into a discrete or continuous set of elementary components, that is, it is expressed as the sum or integral of such components.
2. The response of the system to each component of the input is determined separately.
3. The responses found in (2) are summed or integrated. Because of the additive property of the system, this yields the response of the system to the given input.

[2] The boldface numbers in parentheses refer to the references appended to this paper.

In the following two sections we shall discuss in general terms the salient aspects of the above procedure. The general relationships will be illustrated by application to the special cases of the time and frequency domains.

RESOLUTION

The resolution of a signal $u(t)$ into a set of component signals consists essentially in expressing $u(t)$ in the following form

$$u(t) = \int_C k(t;\lambda)\,U(\lambda)\,d\lambda, \tag{3}$$

where λ is a complex parameter; C is a contour in the λ-plane (generally a straight line parallel to either the imaginary or real axis); $k(t;\lambda)$ is a suitable set of functions of t (representing the component signals in terms of which $u(t)$ is resolved); and $U(\lambda)\,d\lambda$ is a weighting factor which provides a measure of the content in $u(t)$ of those component signals $k(t;\lambda)$ in which the parameter lies between λ and $\lambda + d\lambda$. For convenience in terminology, $k(t;\lambda)$ will be referred to as the *component function*, and $U(\lambda)$ will be called the *spectral function* of $u(t)$ relative to $k(t;\lambda)$.

It will be observed that the case where the $k(t;\lambda)$ form a discrete set can be regarded as a special case of (3) wherein the spectral function $U(\lambda)$ consists of a train of delta-functions (unit impulse functions). The use of delta-functions may be avoided by writing (3) in the Stieltjes integral form.

The familiar Laplace transform relation

$$u(t) = \frac{1}{2\pi j}\int_C e^{st}U(s)\,ds \tag{4}$$

is a special case of (3) corresponding to a component function of the exponential type

$$k(t;\lambda) = \frac{e^{\lambda t}}{2\pi j}. \tag{5}$$

In this relation s plays the role of λ; $U(s)$ is the Laplace transform of $u(t)$; and C is a straight line parallel to the imaginary axis (Bromwich-Wagner contour).

Another simple example is provided by the case where the component signals consist of delta-functions. In this case the component function is of the form

$$k(t;\lambda) = \delta(t - \lambda), \tag{6}$$

where $\delta(t - \lambda)$ is a delta-function, and (3) reduces to

$$u(t) = \int_{-\infty}^{\infty} \delta(t - \lambda)u(\lambda)\,d\lambda. \tag{7}$$

A basic problem arising in connection with (3) is that of expressing the spectral function $U(\lambda)$ in terms of the signal $u(t)$. Owing to the linear character of (3), the expression for $U(\lambda)$ in terms of $u(t)$ is of the general form

$$U(\lambda) = \int_{-\infty}^{\infty} k^{-1}(\lambda; t)u(t)\,dt, \tag{8}$$

where $k^{-1}(\lambda; t)$ denotes the *inverse* of $k(t; \lambda)$. It will be assumed throughout that $k(t; \lambda)$ possesses a unique inverse $k^{-1}(\lambda; t)$. The determination of $k^{-1}(\lambda; t)$—that is, the *inversion* of $k(t; \lambda)$—is a central problem in the theory of integral transforms (3).

A direct relation between the component function $k(t; \lambda)$ and its inverse $k^{-1}(\lambda; t)$ can readily be obtained by applying (8) to a unit impulse signal $u(t) = \delta(t - \xi)$. Thus, on substitution of $\delta(t - \xi)$ for $u(t)$ in (8), the spectral function of $\delta(t - \xi)$ is found to be

$$\begin{aligned} U(\lambda) &= \int_{-\infty}^{\infty} k^{-1}(\lambda; t)\delta(t - \xi)\,dt \\ &= k^{-1}(\lambda; \xi). \end{aligned} \tag{9}$$

Inserting this in (3) and replacing $u(t)$ by $\delta(t - \xi)$, one obtains

$$\int_C k(t; \lambda)k^{-1}(\lambda; \xi)\,d\lambda = \delta(t - \xi), \tag{10}$$

which is the desired relation between $k(t; \lambda)$ and its inverse.

The above relations are well illustrated by the special cases of the exponential and delta-function type component functions. Thus, the inverse of $k(t; \lambda) = e^{\lambda t}/2\pi j$ is

$$k^{-1}(\lambda; t) = e^{-\lambda t}, \tag{11}$$

as can be readily verified by substituting $e^{\lambda t}/2\pi j$ and $e^{-\lambda t}$ for $k(t; \lambda)$ and $k^{-1}(\lambda; t)$ in (10),

$$\frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} e^{\lambda t}e^{-\lambda \xi}\,d\lambda = \delta(t - \xi), \tag{12}$$

and observing that (12) expresses a well-known property of exponential functions.

For the case under consideration, the general relations (3) and (8) reduce to (assuming $u(t) = 0$ for $t < 0$)

$$u(t) = \frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} e^{\lambda t}U(\lambda)\,d\lambda \tag{13}$$

$$U(\lambda) = \int_0^{\infty} e^{-\lambda t}u(t)\,dt, \tag{14}$$

which are the familiar Laplace transformation relations.

The inverse of a component function of the delta-function type,

$k(t;\lambda) = \delta(t-\lambda)$, is

$$k^{-1}(\lambda;t) = \delta(t-\lambda). \tag{15}$$

This is verified by the relation

$$\int_{-\infty}^{\infty} \delta(t-\lambda)\delta(\xi-\lambda)\,d\lambda = \delta(t-\xi), \tag{16}$$

which expresses a well-known property of delta-functions. Correspondingly, the relations (3) and (8) reduce to

$$u(t) = \int_{-\infty}^{\infty} \delta(t-\lambda)U(\lambda)\,d\lambda \tag{17}$$

$$U(\lambda) = \int_{-\infty}^{\infty} \delta(t-\lambda)u(t)\,dt \tag{18}$$
$$= u(\lambda).$$

It is seen that the calculation of the spectral function $U(\lambda)$ relative to a specified component function $k(t;\lambda)$ requires knowledge of the inverse of $k(t;\lambda)$. The inverses of several basic types of component functions are collected below. The inverses of more specialized types may be found in the mathematical literature of integral equations and integral transforms (3, 4). In cases where the determination of the inverse by purely analytical means is not feasible, one has to use either numerical methods (which may require use of machine computation) or integral transform computers of the type described by Wallman (5).

An important type of component function which contains many other types as particular cases is the following:

$$k(t;\lambda) = f(t-\lambda), \tag{19}$$

where $f(t)$ is some specified function. The inverse of $k(t;\lambda)$ is of the same form, that is

$$k^{-1}(\lambda;t) = g(\lambda-t), \tag{20}$$

where $g(\lambda)$ is an as yet undetermined function. To determine $g(\lambda)$, we make use of the general relation between $k(t;\lambda)$ and $k^{-1}(\lambda;t)$ (Eq. 10), which when applied to (19) and (20) gives

$$\int_{-\infty}^{\infty} f(t-\lambda)g(\lambda-\xi)\,d\lambda = \delta(t-\xi). \tag{21}$$

This integral equation can readily be solved for $g(\lambda)$ by applying the Fourier transformation to both sides of (21). The transformed equation reads

$$F(j\omega)G(j\omega)e^{-j\omega\xi} = e^{-j\omega\xi}, \tag{22}$$

where $F(j\omega)$ and $G(j\omega)$ are the Fourier transforms of $f(t)$ and $g(\lambda)$, respectively. Thus

$$G(j\omega) = \frac{1}{F(j\omega)} \tag{23}$$

and consequently

$$g(\lambda) = \mathfrak{F}^{-1}\left\{\frac{1}{F(j\omega)}\right\}, \tag{24}$$

where $\mathfrak{F}^{-1}$ represents the inverse Fourier transformation. The use of this formula is illustrated below by application to a few simple types of component functions.

(*a*) $k(t;\lambda) = \delta(t-\lambda)$. In this case $F(j\omega) = 1$, $G(j\omega) = 1$, and hence $k^{-1}(\lambda;t) = \delta(t-\lambda)$.

(*b*) $k(t;\lambda) = 1(t-\lambda)$ (unit step function). Here $F(j\omega) = 1/j\omega$, $G(j\omega) = j\omega$, and

$$k^{-1}(\lambda;t) = \delta^{(1)}(\lambda - t), \tag{25}$$

where $\delta^{(1)}(\lambda - t)$ denotes a unit doublet function. The expression for the spectral function is

$$U(\lambda) = \int_{-\infty}^{\infty} \delta^{(1)}(\lambda - t)u(t)\,dt \tag{26}$$

$$= \frac{du(\lambda)}{d\lambda}.$$

(*c*) $k(t;\lambda) = 1(t-\lambda) - 1(t-\lambda-1)$ (unit pulse function). For this case, we have

$$F(j\omega) = \frac{1}{j\omega} - \frac{1}{j\omega}e^{-j\omega} \tag{27}$$

$$G(j\omega) = \frac{j\omega}{1 - e^{-j\omega}} \tag{28}$$

$$= j\omega(1 + e^{-j\omega} + e^{-j2\omega} + e^{-j3\omega} + \cdots)$$

and consequently

$$k^{-1}(\lambda;t) = \sum_{n=0}^{\infty} \delta^{(1)}(\lambda - t - n). \tag{29}$$

Substituting this in (8), we find

$$U(\lambda) = \sum_{n=0}^{\infty} u^{(1)}(\lambda - n), \tag{30}$$

where $u^{(1)}(\lambda) = du(\lambda)/d\lambda$.

A known pair of inverse functions, meaning $k(t;\lambda)$ and $k^{-1}(\lambda;t)$, can serve as a prototype for a large number of other such pairs. These can be derived from the prototype in two ways: (*a*) by operating on $k(t;\lambda)$ with a linear operator and transforming the inverse of this operator with $k^{-1}(\lambda;t)$, and (*b*) by changing the variables t and λ in $k(t;\lambda)$ and $k^{-1}(\lambda;t)$. The (*b*) method of derivation is generally more useful than the (*a*) method; its application is governed by the following simple rule:

Let $k^{-1}(\lambda; t)$ be the inverse of $k(t; \lambda)$. Then, the inverse of $k[\alpha(t); \beta(\lambda)]\, d\beta(\lambda)/d\lambda$, where $\alpha(t)$ and $\beta(\lambda)$ are monotone functions, is $k^{-1}[\beta(\lambda); \alpha(t)]\, d\alpha(t)/dt$. The paths of integration in (3) and (8) are transformed in accordance with the relations $\lambda = \beta(\lambda')$ and $t = \alpha(t')$.

A convenient as well as useful illustration of this rule is furnished by the bilateral Laplace transform relations

$$u(t) = \frac{1}{2\pi j} \int_C e^{\lambda t} U(\lambda)\, d\lambda \tag{31}$$

$$U(\lambda) = \int_{-\infty}^{\infty} u(t) e^{-\lambda t}\, dt. \tag{32}$$

In this case $k(t; \lambda) = e^{\lambda t}/2\pi j$; $k^{-1}(\lambda; t) = e^{-\lambda t}$; and C is a line paralleling the imaginary axis and lying within the strip of convergence of $U(\lambda)$.

Now if we replace the variable t in (31) and (32) by $-\log t$, the component function and its inverse become $t^{-\lambda}/2\pi j$ and $-t^{\lambda-1}$, respectively; C is unchanged and the path of integration in (32) is transformed into the positive half of the real axis. Consequently, (31) and (32) become

$$u(t) = \frac{1}{2\pi j} \int_C t^{-\lambda} U(\lambda)\, d\lambda \tag{33}$$

$$U(\lambda) = \int_0^{\infty} t^{\lambda-1} u(t)\, dt, \tag{34}$$

which are known as the Mellin transform relations (3).

Another well-known pair of transforms is associated with a component function of the form

$$k(t; \lambda) = 1(t) \sqrt{\lambda t} J_n(\lambda t), \tag{35}$$

where $1(t)$ is a unit step function and $J_n(\lambda t)$ is a Bessel function of the first kind and order n. The inverse of $k(t; \lambda)$ is of the same form, that is,

$$k^{-1}(\lambda; t) = 1(\lambda) \sqrt{\lambda t} J_n(\lambda t). \tag{36}$$

Correspondingly, the relations between $u(t)$ and $U(\lambda)$ read

$$u(t) = \int_0^{\infty} \sqrt{\lambda t} J_n(\lambda t)\, U(\lambda)\, d\lambda \tag{37}$$

$$U(\lambda) = \int_0^{\infty} \sqrt{\lambda t} J_n(\lambda t)\, u(t)\, dt. \tag{38}$$

In this case $u(t)$ and $U(\lambda)$ are referred to as the Hankel transforms of order n.

There are many other well-known types of integral transforms such as Hilbert transforms, Euler transforms, Whittaker transforms, etc., which are primarily of mathematical interest and appear to have little

use in connection with the resolution of signals. On the other hand, there are many types of component functions that appear to be of practical interest, but have received relatively little attention in the mathematical literature. The use of these component functions is contingent upon the availability of their inverses.

It will be observed that in all of the foregoing discussion it was assumed that λ is a one-dimensional parameter. More generally, however, λ may be a multi-dimensional parameter, in which case (3) takes on the form of a multiple integral (or summation). In particular, Gabor (6) has shown that a two-dimensional mode of resolution into component signals of the form

$$k(t; n, m) = \exp\left[jn\omega_0 t + a(\omega_0 t - 2\pi m)^2\right] \tag{39}$$

where ω_0 and a are constants and n and m collectively play the role of a two-dimensional integral-valued parameter, is useful in certain applications. The main disadvantage of multi-dimensional representations is their lack of uniqueness. More specifically, a multi-dimensional component function $k(t; \lambda)$ and an associated spectral function $U(\lambda)$ uniquely define a signal $u(t)$, but the converse is not true, that is, $u(t)$ and $k(t; \lambda)$ do not uniquely determine $U(\lambda)$. This fact restricts the usefulness of multi-dimensional resolutions.

TRANSFORMATION AND COMPOSITION

It will be recalled that, following the resolution of the input signal $u(t)$ into a set of component signals $k(t; \lambda)$, the determination of the response of the system in question, N, to $u(t)$ is accomplished by finding the responses of N to the components of $u(t)$ and then summing or integrating these responses. In this connection, let $K(t; \lambda)$ be the response of N to $k(t; \lambda)$—both regarded as functions of time involving λ as a parameter. The function $K(t; \lambda)$ will be called a *characteristic function* of N, since it provides a complete characterization of the relation between the input and output of N. For the purposes of our analysis, it will be convenient to regard $K(t; \lambda)$ as a primary characteristic of N, rather than as a derived quantity to be obtained, say, from the solution of the equations governing the behavior of the system.

The special cases of the time and frequency domains will serve to illustrate the concept of characteristic function. Thus, the component function associated with the time domain is a delta-function $k(t; \lambda) = \delta(t - \lambda)$. The corresponding characteristic function—that is, the response to $\delta(t - \lambda)$—is what is usually called the *impulsive response* of N and is denoted by the symbol $W(t, \lambda)$. In mathematical terminology, the impulsive response corresponds essentially to a Green's function (7, 8).

In the case of the frequency domain, the component function has the exponential form expressed by (5). The corresponding characteristic

function, which is the response of N to $e^{\lambda t}/2\pi j$, is written as

$$K(t;\lambda) = H(\lambda;t)\frac{e^{\lambda t}}{2\pi j} \tag{40}$$

and the function $H(\lambda;t)$ is called the *system function* of N (9). In effect, $H(\lambda;t)e^{\lambda t}$ is the response of the system to $e^{\lambda t}$. In the case of a fixed system, the system function $H(\lambda;t)$ is independent of t, that is, is of the form $H(\lambda)$. A popular term for $H(\lambda)$ is "frequency response function."

The response of the system to a specified input $u(t)$ can readily be expressed in terms of a characteristic function $K(t;\lambda)$ and the spectral function of $u(t)$ relative to $k(t;\lambda)$. Thus, because of the linearity of N, the response of N to a differential component $k(t;\lambda)U(\lambda)\,d\lambda$ of $u(t)$, is $K(t;\lambda)U(\lambda)\,d\lambda$. By superposing, that is, integrating, these responses, one obtains the following relation

$$v(t) = \int_C K(t;\lambda)U(\lambda)\,d\lambda, \tag{41}$$

which expresses the response $v(t)$ to an input $u(t)$ in terms of the characteristic function $K(t;\lambda)$ and the spectral function $U(\lambda)$ relative to $k(t;\lambda)$.

The general input-output relation (41) combines and summarizes the three basic steps involved in the determination of the response. A few simple examples will serve to illustrate the connection between (41) and some of the more familiar forms of this relation.

In the particular case of the time domain, $k(t;\lambda) = \delta(t-\lambda)$, $U(\lambda) = u(\lambda)$, and $K(t;\lambda) = W(t,\lambda)$ (the impulsive response). On substitution of these quantities into (41) and noting that λ assumes only real values, one obtains the well-known relation

$$v(t) = \int_{-\infty}^{\infty} W(t,\lambda)u(\lambda)\,d\lambda, \tag{42}$$

which expresses the response of a linear system to an input $u(t)$ in terms of the impulsive response of the system.

Next, consider the case of an exponential component function (Eq. 5). It will be recalled that in this case the spectral function $U(\lambda)$ is the Laplace transform of $u(t)$, and the characteristic function $K(t;\lambda)$ is expressed by (40). Hence, (41) becomes

$$v(t) = \frac{1}{2\pi j}\int_C H(\lambda;t)U(\lambda)e^{\lambda t}\,d\lambda, \tag{43}$$

which is equivalent to the expression given in reference (9). In the particular case of a fixed system, this relation assumes the familiar form

$$v(t) = \frac{1}{2\pi j}\int_C H(\lambda)U(\lambda)e^{\lambda t}\,d\lambda, \tag{44}$$

which constitutes the basis for the Laplace transformation technique of analysis of fixed linear systems.

The above examples indicate that for the particular case of $k(t; \lambda) = \delta(t - \lambda)$, one obtains a set of relations usually associated with the so-called time domain method of analysis. Similarly, for $k(t; \lambda)$ having the exponential form (5) one obtains the well-known frequency domain relations. As an extension of this terminology, it is reasonable to refer to a method of analysis based on the use of a general form of component function $k(t; \lambda)$, as a λ *domain* method. A particular component function gives rise to a particular λ domain. Thus, $k(t; \lambda) = e^{\lambda t}/2\pi j$ gives rise to the frequency domain (λ = complex frequency), and $k(t; \lambda) = \delta(t - \lambda)$ gives rise to the time domain (λ = time).

In dealing with different characteristic functions of a system it frequently becomes necessary to express a characteristic function belonging to a specified domain (that is, associated with a specified component function $k(t; \lambda)$), in terms of a characteristic function belonging to some other domain—generally the time domain. In other words, what is needed in general is a relation between $K(t, \lambda)$ and the impulsive response $W(t, \lambda)$. Such a relation can readily be obtained by using the time domain relation

$$v(t) = \int_{-\infty}^{\infty} W(t, \xi)u(\xi)\, d\xi, \tag{45}$$

where ξ is the variable of integration. Thus, remembering that $K(t; \lambda)$ is the response to $k(t; \lambda)$, it follows at once from (45) that

$$K(t; \lambda) = \int_{-\infty}^{\infty} W(t, \xi)k(\xi; \lambda)\, d\xi, \tag{46}$$

which is the desired result. This relation expresses the characteristic function $K(t; \lambda)$ associated with a specified domain, in terms of the impulsive response of the system and the component function $k(t; \lambda)$ which defines the domain in question.

In order to obtain the inverse relation—that is, an expression for $W(t, \xi)$ in terms of $K(t; \lambda)$—it is sufficient to insert the expression for the spectral function of $\delta(t - \xi)$ in (41). From (9),

$$U(\lambda) = k^{-1}(\lambda; \xi) \tag{47}$$

and consequently

$$W(t, \xi) = \int_C K(t; \lambda)k^{-1}(\lambda; \xi)\, d\lambda, \tag{48}$$

which complements the result expressed by (46).

As an application of these general relations, consider the case of the frequency domain. In this domain, the expressions for $k(t; \lambda)$, $k^{-1}(\lambda; t)$, and $K(t; \lambda)$, are given by (5), (11), and (40), respectively.

Substituting these in (46) and (48), one obtains the following relations

$$H(\lambda; t) = \int_{-\infty}^{\infty} W(t, \xi) e^{-\lambda(t-\xi)}\, d\xi \tag{49}$$

and

$$W(t, \xi) = \frac{1}{2\pi j} \int_c H(\lambda; t) e^{\lambda(t-\xi)}\, d\lambda, \tag{50}$$

which are equivalent to the corresponding relations given in reference (**8**).

In analyzing a given signal transmission system it is convenient to conduct the analysis in a domain in which the relation (41) between the input and output assumes a simple form. It is readily seen that the simplest form of this relation results when the characteristic function $K(t; \lambda)$ is of the form

$$K(t; \lambda) = A(\lambda) k(t; \lambda), \tag{51}$$

where $A(\lambda)$ is some function of λ. For, in this case the relation between $v(t)$ and $u(t)$ becomes

$$v(t) = \int_c A(\lambda) U(\lambda) k(t; \lambda)\, d\lambda, \tag{52}$$

which is equivalent to the simple algebraic relation

$$V(\lambda) = A(\lambda) U(\lambda) \tag{53}$$

between the spectral functions of $v(t)$ and $u(t)$ relative to $k(t; \lambda)$.

In the case of a fixed signal transmission system, the form (51) is obtained in conjunction with the exponential type component function $k(t; \lambda) = e^{\lambda t}/2\pi j$. Thus, the characteristic function corresponding to this component function reads

$$K(t; \lambda) = H(\lambda) \frac{e^{\lambda t}}{2\pi j}, \tag{54}$$

where $H(\lambda)$ is the frequency response function. Correspondingly, (53) becomes

$$V(\lambda) = H(\lambda) U(\lambda), \tag{55}$$

which is the familiar relation between the Laplace transforms of the input and output of a fixed system.

In the case of an amplitude modulating system, the component function resulting in the desired form (51) is $k(t; \lambda) = \delta(t - \lambda)$. With this component function, one has

$$K(t; \lambda) = G(\lambda) \delta(t - \lambda), \tag{56}$$

where $G(\lambda)$ is the instantaneous gain of the amplitude modulator. Correspondingly, (53) reduces to the algebraic relation

$$v(\lambda) = G(\lambda) u(\lambda). \tag{57}$$

It is not always possible to find a component function in terms of which the characteristic function of a given system assumes the desired form (51). The problems associated with the existence and the determination of such component functions belong in the mathematical theory of spectral resolution of linear operators (10).

FUNCTION SPACE INTERPRETATION

The various relations established in the preceding sections can be conveniently interpreted in geometrical terms by the use of the function space approach (11, 12, 13, 15). In employing this approach, a signal $u(t)$ is represented as a vector $\mathbf{u}$ in an infinite-dimensional function space (signal space). In general, a function space is nonseparable, that is, has a nondenumerable infinitude of dimensions. However, when the signals under consideration belong to a certain class of functions such as the class of functions of integrable square (L_2 class), or the class of continuous functions defined on a finite interval, or the class of periodic functions, etc., one can represent the signals in question as points in a function space having a denumerable infinity of dimensions. In particular, signals belonging to the L_2 class can be represented as points in a Hilbert space.

Now let Σ be the signal space, that is, a function space representing the class of signals under consideration. From a geometric point of view, a set of component signals $k(t;\lambda)$ constitutes a *basis* or a coordinate system in Σ. The resolution of a signal $u(t)$ in the form

$$u(t) = \int_C k(t;\lambda)\, U(\lambda)\, d\lambda \tag{58}$$

corresponds to a decomposition of $u(t)$ along the axes of the coordinate system $k(t;\lambda)$. The values of $U(\lambda)$ constitute the coordinates of $u(t)$ with respect to this coordinate system.

An operation with a transmission system N on a signal $u(t)$ has the effect of transforming the basis $k(t;\lambda)$ into a different basis $K(t;\lambda)$, leaving invariant the coordinates of $u(t)$ with respect to $k(t;\lambda)$, that is, $U(\lambda)$. This gives rise to the relation

$$v(t) = \int_C K(t;\lambda)\, U(\lambda)\, d\lambda, \tag{59}$$

which expresses the response of N to $u(t)$ in terms of a decomposition of $v(t)$ along the axes of the transformed coordinate system $K(t;\lambda)$.

A set of vectors (signals) $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \cdots$ spans a linear subspace (manifold) M in Σ which consists of all vectors of the form $c_1\mathbf{u}_1 + c_2\mathbf{u}_2 + \cdots$ where $c_1, c_2, \cdots$ are arbitrary coefficients. For example, in the case of the class of signals of integrable square, Σ is a Hilbert space. A subclass of signals of this class that consists of signals which do not contain

frequencies between, say, f_1 and f_2 cycles per second, corresponds to a linear manifold in Σ.

A vector $\mathbf{u}$ may be projected on a manifold M in Σ in much the same manner as a vector in the three-dimensional Euclidean space is projected on a plane or a straight line. A projection is specified by its *range* (the manifold M on which $\mathbf{u}$ is projected), and by the "direction" of projection (a manifold M', complementary to M, *along* which $\mathbf{u}$ is projected on M). For example, in the three-dimensional space, $\mathbf{u}$ may be projected on a plane P along a line L, or on a line L along a plane P. (P and L constitute a pair of complementary manifolds in this space.) A projection (regarded as an operation on $\mathbf{u}$) is idempotent, that is, yields the same result upon repetition.

The projection of the signal space Σ on a manifold M along M' may be interpreted in physical terms as the filtering of the class of signals representing Σ with an *ideal filter* N which passes without distortion (or delay) all signals belonging to M and rejects all those belonging to M' (14, 2). For example, consider an ideal low-pass filter which passes all frequencies below, say, f_0 cps. and rejects all those above f_0 cps. Let M_0 be the manifold representing signals which do not contain frequencies above f_0 cps., and let M_0' be the manifold of signals not containing frequencies below f_0 cps. Then, in geometrical terms, the filter in question has the effect of projecting the signal space Σ on M_0 along M_0'.

The idempotent property of projections and the equivalence between the latter and the process of filtration with ideal filters, permits defining an ideal linear filter in the following manner: *A signal transmission system N is an ideal linear filter if it is idempotent, that is, if it is equivalent to a tandem combination of two systems each of which is identical with N.*

In general, an ideal filter is not a physically realizable system inasmuch as its impulsive response $W(t, \lambda)$ does not, in general, vanish for $t < \lambda$. However, in most cases one can approximate to $W(t - \beta, \lambda)$, where β is a sufficiently large constant, by a realizable system. In other words, one can approximate an ideal filter, to within a constant time delay, by a physical filter.

The concept of ideal filter has been used heretofore chiefly in connection with *fixed* filters in the frequency domain (16). The more general definition formulated above extends this concept to *variable* filters. (A still more general definition which applies to both linear and nonlinear filters is given in (1).) It is readily verified that an ideal filter in the frequency domain is a special case of the more general type just defined. Thus consider, for example, an ideal low-pass filter in the frequency domain. Such a filter has a gain of unity (and zero phase shift) over the pass-band, and zero gain over the stop-band. It is obvious that a

tandem combination of two such filters is equivalent to one of them, and hence the filter in question is idempotent.

In the particular case of the time domain, an ideal filter assumes the form of a switch. This follows at once from the fact that a tandem (series) connection of two identical switches is equivalent to only one of these switches, which means that a switch is an idempotent system. The "pass-band" of a switch is the time interval during which it is closed, and the "stop-band" is the time interval over which it is open.

Ideal filters have many uses, of which only a few have thus far been exploited in practice. In particular, it can readily be shown that any signal transmission system (fixed or variable) may be approximated (to within a constant time delay) by a weighted parallel-series combination of ideal filters. Furthermore, by using ideal filters it is possible to separate signals which occupy the same frequency band and overlap in the time domain, provided only the spectral functions of the signals do not overlap in some λ domain. In function space terms, this means that the manifolds spanned in the signal space by the signals in question should be *disjoint*, that is, should not intersect each other except at the origin. Signals having this property can be transmitted simultaneously (multiplexed) and separated at the receiving end by using appropriate ideal filters. For example, consider two sets of signals $\{u_1(t)\}$ and $\{u_2(t)\}$ which span disjoint manifolds M_1 and M_2 in Σ, and assume that the signals are transmitted simultaneously, so that one has the sum $\{u_1(t) + u_2(t)\}$ at the receiving end. It is clear that one can extract the $\{u_1(t)\}$ set, say, from $\{u_1(t) + u_2(t)\}$ by filtering the sum with an ideal filter which passes without distortion all signals belonging to M_1 and rejects all those belonging to M_2. It will be observed that the conventional methods of multiplexing based on the use of either time or frequency division, are special cases of the more general approach based on the use of signals which span disjoint manifolds in the signal space.

OPERATIONAL SYMBOLISM

In dealing with signal transmission systems it is frequently convenient to employ an operational form of symbolism which is based on treating characteristic functions as operators. Thus, a characteristic function $K(t;\lambda)$ may be regarded as a linear operator which operates on a given input $u(t)$ and yields the corresponding output $v(t)$. The symbolism in question amounts essentially to writing the relation

$$v(t) = \int_c K(t;\lambda)\,U(\lambda)\,d\lambda \tag{60}$$

in an abbreviated form

$$v(t) = K(t;q)u(t), \tag{61}$$

where the use of q in place of λ serves to indicate that $K(t;q)$ should be treated as an operator. When no ambiguity results, this relation

may be written more simply in the usual symbolic form

$$v = Ku. \tag{62}$$

Taken together, (60) and (61) establish an equivalence between the statements "the response of a system defined by a characteristic function $K(t;\lambda)$ to an input $u(t)$" and "the result of operating with an operator $K(t;q)$ on $u(t)$."

A special and particularly important class of operators obtains in the case of the frequency domain. In this domain, $k(t;\lambda)$ and $K(t;\lambda)$ are expressed by (5) and (40). Substituting these expressions in (60) and (61) and combining the results, one gets the following relation

$$H(q;t)\frac{e^{qt}}{2\pi j}u(t) = \frac{1}{2\pi j}\int_C H(\lambda;t)U(\lambda)e^{\lambda t}\,d\lambda, \tag{63}$$

which for convenience is abbreviated to

$$H(q;t)u(t) = \frac{1}{2\pi j}\int_C H(\lambda;t)U(\lambda)e^{\lambda t}\,d\lambda. \tag{64}$$

It should be understood that this relation is nothing more than a definition; that is, the right-hand member in this relation serves to define the left-hand member.

In order to distinguish between general operators of the form $K(t;q)$ and the special class of operators $H(q;t)$ defined above, it is expedient to use some symbol other than q in connection with the latter. For reasons to become evident shortly, the proper symbol to use in place of q is p. Making this change in (64) and furthermore replacing λ with $j\omega$, we obtain the following defining relation

$$H(p;t)u(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(j\omega;t)U(j\omega)e^{j\omega t}\,d\omega, \tag{65}$$

where $U(j\omega)$ is the Fourier transform of $u(t)$.

The operator $H(p;t)$ is called a *time-dependent Heaviside operator* (17, 18). The basis for calling it thus is provided by the fact that in the particular case of fixed systems, (65) reduces to

$$H(p)u(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(j\omega)U(j\omega)e^{j\omega t}\,d\omega, \tag{66}$$

which will be recognized as the definition of an ordinary Heaviside operator $H(p)$. It will also be noted that when $H(p;t)$ is a polynomial in p, p plays the role of d/dt and $H(p;t)$ reduces to a differential operator.

Referring to (65), it will be observed that the variable t in $H(j\omega;t)$ plays the role of a parameter. Consequently, in evaluating the result of operating with $H(p;t)$ on $u(t)$, one can treat $H(p;t)$ as if it were an ordinary Heaviside operator involving t as a parameter.

Product Relation

A very important relation in the theory of operators is the so-called *product relation*, which arises in connection with successive operations on an operand with two or more operators. Thus consider a relation of the form

$$v(t) = K_2(t; q)\{K_1(t; q)u(t)\} \tag{67}$$

or more simply

$$v = K_2\{K_1u\}. \tag{68}$$

By definition, the *product* of $K_2(t; q)$ and $K_1(t; q)$ is an operator $K_3(t; q)$ such that

$$K_3(t; q)u(t) = K_2(t; q)\{K_1(t; q)u(t)\} \tag{69}$$

for all $u(t)$. This is expressed symbolically by the relation

$$K_3(t; q) = K_2(t; q)*K_1(t; q) \tag{70}$$

or more simply

$$K_3 = K_2*K_1. \tag{71}$$

The symbol * serves to distinguish between the algebraic product of $K_2(t; q)$ and $K_1(t; q)$, and the product in the sense of Eq. 70. When no confusion results, (71) may be written more simply in the form (commonly used in the mathematical literature)

$$K_3 = K_2K_1. \tag{72}$$

The operation * has the usual distributive and associative properties of an algebraic product, but, in general, is not commutative. Thus, we have the following identities:

$$K_a*(K_b + K_c) \equiv K_a*K_b + K_a*K_c \tag{73}$$

$$K_a*(K_b*K_c) \equiv (K_a*K_b)*K_c \tag{74}$$

but not, in general, $K_a*K_b \equiv K_b*K_a$.

Physically, the product of K_2 and K_1 may be interpreted as follows. Let K_1 and K_2 be the characteristic functions (relative to a λ domain) of two systems N_1 and N_2. Then, K_2*K_1 represents the characteristic function (relative to the same domain) of the series connection of N_1 and N_2, with N_1 preceding N_2. This is illustrated in Fig. 2.

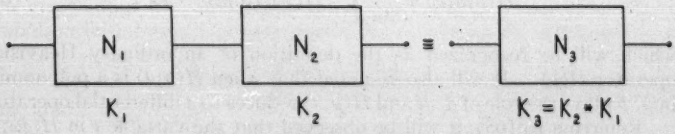


FIG. 2. Physical interpretation of the product relation.

An explicit expression for the product can readily be obtained by referring to Fig. 2 and noting that for the particular case where the input

$u(t)$ is equal to $k(t;\lambda)$, the response of N_1 to this input is $K_1(t;\lambda)$, and the response of N_2 to $K_1(t;\lambda)$ should be equal to $K_3(t;\lambda)$. In operational form, the response of N_2 to $K_1(t;\lambda)$ is given by $K_2(t;q)K_1(t;\lambda)$, and hence the expression for $K_3(t;\lambda)$ becomes

$$K_3(t;\lambda) = K_2(t;q)K_1(t;\lambda). \tag{75}$$

This relation gives an explicit meaning to the symbolical relations (70) and (71).

The product relation (75) assumes familiar forms in the special cases of the time and frequency domains. Thus in the case of the time domain, the defining relation (61) reduces to

$$W(t,q)u(t) = \int_{-\infty}^{\infty} W(t,\lambda)u(\lambda)\,d\lambda \tag{76}$$

and consequently (75) becomes

$$W_3(t,\lambda) = \int_{-\infty}^{\infty} W_2(t,\lambda')W_1(\lambda',\lambda)\,d\lambda', \tag{77}$$

where λ' is the variable of integration. The relation expressed by (77) is known as Volterra's rule of composition (19).

In the particular case of fixed systems, the impulsive response is of the form $W(t-\lambda)$ and (77) reduces to

$$W_3(t-\lambda) = \int_{-\infty}^{\infty} W_2(t-\lambda')W_1(\lambda'-\lambda)\,d\lambda', \tag{78}$$

which is the familiar convolution relation.

In the frequency domain, the operational symbolism is defined by (65). By using this relation in conjunction with (75), one readily finds the following expression for the product in terms of the system functions

$$H_3(\lambda;t) = H_2(p+\lambda;t)H_1(\lambda;t), \tag{79}$$

where $H_1(\lambda;t)$ plays the role of the operand. For a fixed system, the product relation (79) reduces to the familiar algebraic relation

$$H_3(\lambda) = H_2(\lambda)H_1(\lambda). \tag{80}$$

Inverse Systems and Operators

For convenience in terminology, a signal transmission system N which is such that the response of N to any input is identical with the input, will be referred to as a *unit* system. Evidently, the characteristic function of a unit system relative to $k(t;\lambda)$ is equal to $k(t;\lambda)$, that is,

$$K(t;\lambda) = k(t;\lambda). \tag{81}$$

A characteristic function of this form will be called a *unit* characteristic function, and correspondingly an operator of the form $K(t;q) = k(t;q)$ will be referred to as the *identity* operator.

Two systems N and N^{-1} are said to be *inverses* of one another if a tandem connection of N and N^{-1} (in any order) is equivalent to a unit system. If N possesses an inverse it is called *non-singular*.

Let $K(t;\lambda)$ (or simply K) be the characteristic function of N relative to $k(t;\lambda)$ (or k). The corresponding characteristic function of N^{-1}, if it exists, is denoted by $K^{-1}(t;\lambda)$ (or K^{-1}) and is called the *inverse* of $K(t;\lambda)$. The relation between K and K^{-1} is

$$K*K^{-1} = K^{-1}*K = k \tag{82}$$

or more explicitly

$$K(t;q)K^{-1}(t;\lambda) = K^{-1}(t;q)K(t;\lambda) = k(t;\lambda). \tag{83}$$

In the time domain, the unit characteristic function is the delta-function $\delta(t-\lambda)$ and hence (82) becomes

$$\int_{-\infty}^{\infty} W(t,\lambda')W^{-1}(\lambda',\lambda)\,d\lambda' = \delta(t-\lambda), \tag{84}$$

where W and W^{-1} are the impulsive responses of N and N^{-1}, respectively.

In the frequency domain, the system function of a unit system is equal to unity. Consequently, if H is the system function of N, and H^{-1} (the inverse system function) is the system function of N^{-1}, then in view of (82) and (79), we have

$$H*H^{-1} = H^{-1}*H = 1 \tag{85}$$

or more explicitly

$$H(p+\lambda;t)H^{-1}(\lambda;t) = H^{-1}(p+\lambda;t)H(\lambda;t) = 1. \tag{86}$$

In the particular case of fixed systems, this relation reduces to

$$H(\lambda)H^{-1}(\lambda) = 1 \tag{87}$$

or

$$H^{-1}(\lambda) = \frac{1}{H(\lambda)}, \tag{88}$$

which expresses simply the well-known fact that if $H(\lambda)$ is the frequency response function of a fixed system N, then the frequency response function of the inverse of N is equal to the algebraic reciprocal of $H(\lambda)$.

STABILITY

An important aspect of the behavior of a signal transmission system is that of system stability. We shall confine ourselves here to defining a stable system and stating a general criterion of stability.

A transmission system N is *stable* if the response of N to every bounded input is bounded. N is *unstable* if there is some bounded input which produces an unbounded output.

It can readily be shown that a necessary and sufficient condition for stability is that the magnitude of the impulsive response $W(t, \lambda)$ should be integrable with respect to λ for all $t > t_0$, that is,

$$\int_{-\infty}^{t} |W(t, \lambda)| \, d\lambda < \infty, \qquad \text{for } t > t_0, \tag{89}$$

where t_0 is a specified instant of time which marks the beginning of the period of observation of the behavior of the system.

It is generally difficult to formulate a necessary and sufficient condition for stability in terms of a characteristic function other than $W(t, \lambda)$. For instance, in terms of the system function $H(\lambda; t)$ it can readily be shown (20) that a necessary condition for stability is that $H(\lambda; t)$ should be regular and bounded in the right-half and on the imaginary axis of the λ-plane for all $t > t_0$. This condition, however, is not sufficient, although for most practical purposes it may be regarded as such. The precise formulation of a necessary and sufficient condition in terms of $H(\lambda; t)$ is much more difficult than in terms of the impulsive response $W(t, \lambda)$.

Acknowledgments

The author is indebted to J. B. Russell, J. R. Ragazzini, and L. H. O'Neill of the Electrical Engineering Department, Columbia University, for many stimulating discussions.

REFERENCES

(1) L. A. ZADEH AND K. S. MILLER, "Generalized Ideal Filters" (to be published in *J. Appl. Phys.*).
(2) L. A. ZADEH, "On the Theory of Filtration of Signals," to be published in *Zeit. Angew. Math. Phys.*
(3) C. J. TRANTER, "Integral Transforms in Mathematical Physics," New York, John Wiley & Sons, Inc., 1951.
(4) W. SCHMEIDLER, "Lineare Integralgleichungen," Leipzig, Geest & Portig K.-G., 1950.
(5) H. WALLMAN, "An Electronic Integral-Transform Computer and the Practical Solution of Integral Equations," JOUR. FRANKLIN INST., Vol. 250, pp. 45–61 (1950).
(6) D. GABOR, "Theory of Communication," *J. I. E. E.*, Vol. 93, Pt. III, pp. 429–457 (1946).
(7) S. A. SCHELKUNOFF, "Applied Mathematics for Engineers and Scientists," New York, D. Van Nostrand Co., Inc., 1948.
(8) L. A. ZADEH, "The Determination of the Impulsive Response of Variable Networks," *J. Appl. Phys.*, Vol. 21, pp. 642–644 (1950).
(9) L. A. ZADEH, "Frequency Analysis of Variable Networks," *Proc. I. R. E.*, Vol. 38, pp. 291–299 (1950).
(10) "Proceedings of the Symposium on Spectral Theory and Differential Problems," Oklahoma, Oklahoma A. and M., 1951.
(11) H. WEYL, "Theory of Groups and Quantum Mechanics," New York, Dover Publications, Inc., 1949.
(12) F. J. MURRAY, "Linear Transformations in Hilbert Space," Princeton, Princeton University Press, 1941.

(13) C. E. SHANNON, "Communication in the Presence of Noise," *Proc. I. R. E.*, Vol. 37, pp. 10–21 (1949).
(14) L. A. ZADEH, "Theory of Filtering of Signals in a λ Domain," presented New York Meeting Amer. Math. Soc., April 27, 1951. *Bull. A. M. S.*, Vol. 57, p. 278 (1951).
(15) L. A. ZADEH, "Some Basic Problems in Communication of Information," to be published in *Trans. N. Y. Acad. Sci.*
(16) E. A. GUILLEMIN, "Communication Networks," Vol. II, New York, John Wiley & Sons, Inc., 1935.
(17) L. A. ZADEH, "Time-Dependent Heaviside Operators," *J. Math. and Phys.*, Vol. 30, pp. 73–78 (1951).
(18) L. A. ZADEH, "Circuit Analysis of Linear Varying-Parameter Networks," *J. Appl. Phys.*, Vol. 21, pp. 1171–1177 (1950).
(19) V. VOLTERRA, "Theory of Functionals," London, Blackie and Sons, 1930.
(20) L. A. ZADEH, "On Stability of Linear Varying-Parameter Systems," *J. Appl. Phys.*, Vol. 22, pp. 402–405 (1951).

FIELD EXPANDABILITY IN NORMAL MODES FOR A MULTILAYERED RECTANGULAR OR CIRCULAR WAVE-GUIDE

BY

J. VAN BLADEL[1]

SUMMARY

The purpose of this article is to prove that the field in a multilayered rectangular or circular wave-guide can be expressed as a linear sum of normal modes, and to show how to obtain the field excited by a given current distribution in the guide.

The study of electromagnetic phenomena in wave-guides is simplified by expansion of the field in an infinite series of partial waves termed normal modes. It is indeed possible to calculate, for example, from a distribution of sources, the coefficients of each of these partial waves and to find the field at any point by summation of the series, a process which is simplified by the following considerations.

Let z be the axial direction of the cylindrical guide. The normal modes have been taken by numerous authors to be waves with a common z dependence of the form e^{ihz} for the six coordinates of the field $(\bar{E}, \bar{H})$. Each of these waves must satisfy Maxwell's equations and the conditions at the boundaries of the guide, metallic walls or dielectric interfaces. As a result of this, h is allowed to take an infinite number of discrete values, given as functions of the frequency by the "characteristic equation." Each value defines a mode, and each mode has a cut-off frequency below which it is damped (h imaginary), and above which it is propagated (h real). Only a finite number of modes are propagated at any given frequency. An approximation for the field can be found by limiting the summation of the series for the propagated waves. The accuracy of this approximation increases with the distance of the field point from the sources, and is sufficient for the experimental conditions of practical devices. Parameters are often chosen so as to have only one propagated mode, the "lowest one," and all but the first term of the series are discarded.

It is, of course, essential to show that any wave-guide field can actually be split into a sum of so-defined exponential waves. This proof has been given (1)[2] for a wave-guide of arbitrary cross-section filled with a homogeneous dielectric, the modes of which can be classified as E waves ($H_z = 0$) and H waves ($E_z = 0$). The ensemble of the E_z's and that of the H_z's of the various modes (eigen-functions of the wave equation in the xy plane) form two complete and orthogonal sets.

[1] Manufacture Belge de Lampes Electriques, Brussels, Belgium.

[2] The boldface numbers in parentheses refer to the references appended to this paper.

The problem can be reduced to a scalar one and this completeness property is the basis of the proof.

The situation is more intricate for a non-homogeneous dielectric, where the exponential z waves have both E_z and H_z components. The only orthogonality property is a vectorial one (2), namely, that for two modes m and n,

$$\int_A \bar{I}_z \cdot (\bar{E}_{tm} \times \bar{H}_{tn}) \cdot dS = \delta_{mn}, \tag{1}$$

where t denotes a component on the plane (xy) of the cross-section with area A. This property insures uniqueness of the expansion, and furnishes its coefficients but does not prove expandability. The purpose of the present note is to give this proof for two dielectric dispositions of great practical importance.

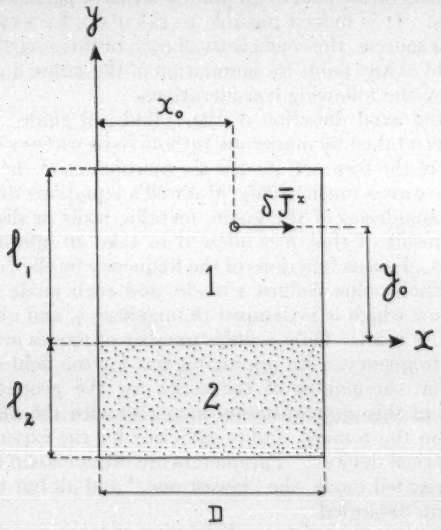


FIG. 1.

Consider first the rectangular cross-section of Fig. 1, filled with two lossless dielectrics and bounded by perfectly conducting walls. Assume phenomena to be harmonic in time. The first step in the proof is to show that the field due to an element of current oriented along one of the axes Ox, Oy or Oz can be expanded as a sum of normal modes. Let

$$\delta J^z = \delta(x - x_0) \cdot \delta(y - y_0) \cdot \delta(z - z_0) \cdot \bar{I}_z \tag{2}$$

be the current element at x_0, y_0, z_0 flowing parallel to Ox. Defining the infinite Fourier transform as

$$[f(h)] = \int_{-\infty}^{+\infty} e^{ihz} \cdot f(z) \cdot dz, \tag{3}$$

it is found that in medium i ($i = 1, 2$):

$$\nabla^2_{xy}[E_z] + \left(\frac{\omega^2}{c_i^2} - h^2\right) \cdot [E_z] = -\frac{h}{\epsilon\omega} \cdot e^{ihz_0} \cdot \delta(y - y_0) \cdot \frac{\partial}{\partial x} \delta(x - x_0) \tag{4}$$

$$\nabla^2_{xy}[H_z] + \left(\frac{\omega^2}{c_i^2} - h^2\right) \cdot [H_z] = e^{ihz_0} \cdot \delta(x - x_0) \cdot \frac{\partial}{\partial y} \delta(y - y_0) \tag{5}$$

a result obtained by taking the Fourier transform of the wave equations with second members satisfied by E_z and H_z. Boundary conditions valid for all values of z must be satisfied by the Fourier transforms so that

(*a*) $[E_z] = [H_n] = 0$ at a metallic wall with normal n. The transformed Maxwell equations make it possible to express $[E_x]$, $[E_y]$, $[H_x]$, and $[H_y]$ as functions of $[E_z]$ and $[H_z]$, and condition $[H_n] = 0$ is seen therefrom to be equivalent to $\frac{\partial}{\partial n}[H_z] = 0$.

(*b*) $[E_z]$, $[H_z]$, $[E_z]$, and $[H_z]$ are continuous at a dielectric interface, the latter two transforms being furthermore, respectively, equal to

$$[E_{x_i}] = \frac{ih}{k_i^2} \cdot \frac{\partial[E_z]}{\partial x} - \frac{i\omega\mu_i}{k_i^2} \cdot \frac{\partial[H_z]}{\partial y} \tag{6}$$

$$[H_{x_i}] = \frac{ih}{k_i^2} \cdot \frac{\partial[H_z]}{\partial x} + \frac{i\omega\epsilon_i}{k_i^2} \cdot \frac{\partial[E_z]}{\partial y}, \tag{7}$$

where

$$k_i^2 = \frac{\omega^2}{c_i^2} - h^2.$$

The above z-coordinate transforms are functions of h, x, y. Application of finite Fourier transforms to them will eliminate the x-coordinate. Finite sine and cosine Fourier transforms are defined by

$$S_n f = \int_0^D f(x) \cdot \sin n\pi \frac{x}{D} \cdot dx \tag{8}$$

$$C_n f = \int_0^D f(x) \cdot \cos n\pi \frac{x}{D} \cdot dx \tag{9}$$

while the original function is restored by inversion formulas

$$f(x) = \frac{2}{D} \cdot \sum_n S_n \cdot \sin n\pi \frac{x}{D} = \frac{2}{D} \cdot \sum_n C_n \cdot \cos n\pi \frac{x}{D}. \tag{10}$$

The finite transforms have interesting operational properties (3) relating the transform of a derivative to the transform of the function itself. For example:

$$S_n f' = -\frac{n\pi}{D}\cdot C_n f, \tag{11}$$

$$C_n f' = \frac{n\pi}{D}\cdot S_n f - f(0) + (-1)^n\cdot f(D), \tag{12}$$

$$S_n f'' = -\left(\frac{n\pi}{D}\right)^2\cdot S_n f + \frac{n\pi}{D}\cdot[f(0) - (-1)^n f(D)], \tag{13}$$

$$C_n f'' = -\left(\frac{n\pi}{D}\right)^2\cdot C_n f - f'(0) + (-1)^n\cdot f'(D). \tag{14}$$

The sine and cosine transforms of the second derivatives are well suited for $[E_z]$ and $[H_z]$, respectively, because of the boundary conditions at the walls $x = 0$ and $x = D$ which cancel the last two terms of (13) or (14). Taking the sine transform of (4) and the cosine transform of (5) yields

$$\frac{\partial^2}{\partial y^2} S_n[E_z] + S_n[E_z]\cdot\left[k_i^2 - \left(\frac{n\pi}{D}\right)^2\right]$$
$$= -\frac{h}{\epsilon\omega}\cdot e^{ihz_0}\cdot\delta(y - y_0)\cdot S_n\frac{\partial}{\partial x}[\delta(x - x_0)]$$
$$= \frac{h}{\epsilon\omega}\cdot e^{ihz_0}\cdot\delta(y - y_0)\cdot\cos\frac{n\pi}{D}x_0 \tag{15}$$

$$\frac{\partial^2}{\partial y^2} C_n[H_z] + C_n[H_z]\cdot\left[k_i^2 - \left(\frac{n\pi}{D}\right)^2\right]$$
$$= -e^{ihz_0}\cdot\frac{\partial}{\partial y}[\delta(y - y_0)]\cdot C_n\delta(x - x_0)$$
$$= -e^{ihz_0}\cdot\frac{\partial}{\partial y}[\delta(y - y_0)]\cdot\cos\frac{n\pi}{D}x_0. \tag{16}$$

Equations 15 and 16, in which the x-derivatives have disappeared, express the y-dependence of $S_n[E_z]$, $C_n[H_z]$, which are functions of y, h, n. They will be solved with the aid of the boundary conditions, remembering that all conditions true for all values of x between 0 and D are valid for the transforms. In consequence:

(*a*) $S_n[E_z]$ and $\frac{\partial}{\partial y} C_n[H_z]$ vanish at the metallic walls $y = l_1$ and $y = l_2$.

(*b*) $S_n[E_z]$ and $C_n[H_z]$ are continuous at the dielectric interface as

well as $C_n[E_z]$ and $S_n[H_z]$, which by taking the cosine and sine transforms of (6) and (7), respectively, are found to be equal to

$$C_n[E_z] = \frac{1}{k_i^2}\left[ih \cdot \frac{n\pi}{D} \cdot S_n[E_z] - i\omega\mu_i \frac{\partial}{\partial y} C_n[H_z]\right] \tag{17}$$

$$S_n[H_z] = \frac{1}{k_i^2}\left[i\omega\epsilon_i \cdot \frac{\partial}{\partial y} S_n[E_z] - ih \cdot \frac{n\pi}{D} \cdot C_n[H_z]\right]. \tag{18}$$

Equations 15 and 16 have a second member containing δ functions or their derivatives. One can get rid of this second member, which vanishes everywhere but in y_0, by integrating both members between $y_0 - \epsilon$ and $y_0 + \epsilon$. This procedure replaces the second member by the following conditions in y_0:

(*a*) that $S_n[E_z]$ be continuous and $\frac{\partial}{\partial y} S_n[E_z]$ have a discontinuity equal to $\frac{h}{\epsilon\omega} \cdot e^{ihz_0} \cdot \frac{n\pi}{D} \cdot \cos\frac{n\pi}{D} x_0$.

(*b*) that $\frac{\partial}{\partial y} C_n[H_z]$ be continuous and $C_n[H_z]$ have a discontinuity equal to $-e^{ihz_0} \cdot \cos\frac{n\pi}{D} x_0$.

Solving (15) and (16), equations without second member, yields, by taking boundary conditions at the walls into account,

(*a*) for $l_1 > y > y_0$

$$S_n[E_z] = C_1 \cdot \sin\frac{u_1}{l_1}(y - l_1) \tag{19}$$

$$C_n[H_z] = D_1 \cdot \cos\frac{u_1}{l_1}(y - l_1). \tag{20}$$

(*b*) for $y_0 > y > 0$

$$S_n[E_z] = E_1 \cdot \sin\frac{u_1}{l_1}(y - l_1) + F_1 \cdot \cos\frac{u_1}{l_1}(y - l_1) \tag{21}$$

$$C_n[H_z] = G_1 \cdot \sin\frac{u_1}{l_1}(y - l_1) + H_1 \cdot \cos\frac{u_1}{l_1}(y - l_1). \tag{22}$$

(*c*) for $0 > y > -l_2$

$$S_n[E_z] = C_2 \cdot \sin\frac{u_2}{l_2}(y + l_2) \tag{23}$$

$$C_n[H_z] = D_2 \cdot \cos\frac{u_2}{l_2}(y + l_2) \tag{24}$$

with the notation

$$\frac{\omega^2}{c_i^2} - h^2 - \left(\frac{n\pi}{D}\right)^2 = u_i^2. \tag{25}$$

The eight coefficients $C_1 \cdots D_2$ can be determined by the eight boundary conditions in $y = y_0$ and $y = 0$, which constitute a non-homogeneous system of eight linear equations. This system will not be written down for the sake of brevity. It suffices to say that each coefficient, which is a function of w, h, n, is equal to the quotient of two determinants of the eighth order. After expansion and simplifications, the denominator Q of quotient $\frac{P}{Q}$ is found to be

$$Q = \left(\frac{\mu_1 u_1 tgu_1}{l_1 k_1^2} + \frac{\mu_2 u_2 tgu_2}{l_2 k_2^2}\right)\cdot\left(\frac{\epsilon_1 u_1}{k_1^2 l_1 tgu_1} + \frac{\epsilon_2 u_2}{k_2^2 l_2 tgu_2}\right) + \left(\frac{hn\pi}{D}\right)^2 \cdot \left(\frac{1}{k_1^2} - \frac{1}{k_2^2}\right) \tag{26}$$

and is the same for all eight coefficients. The all-important point is that $Q = 0$ furnishes the characteristic equation of the modes in the wave-guide of Fig. 1 (4).

Substitution of the values P/Q in (19) to (24) yields the full analytical expression for $S_n[E_z]$ and $C_n[H_z]$, from which E_z and H_z can be restored by two inversion operations as

$$E_z(x, y, z) = \frac{1}{\pi D} \sum_n \sin \frac{n\pi x}{D} \cdot \int_{-\infty}^{+\infty} e^{-ihz} \cdot S_n(h, \omega)[E_z] \cdot dh \tag{27}$$

$$H_z(x, y, z) = \frac{1}{\pi D} \sum_n \cos \frac{n\pi x}{D} \cdot \int_{-\infty}^{+\infty} e^{-ihz} \cdot C_n(h, \omega)[H_z] \cdot dh. \tag{28}$$

$S_n[E_z]$ and $C_n[H_z]$ are functions of h, both directly and through the auxiliary variables u_i (25). They all have Q as denominator, and no branch points. The only possible branch points are, indeed, the values h_b for which $u_i = 0$ (see 25), because u_i goes over into $-u_i$ when h describes a small circle around these points. However, a consideration of the quotients of determinants mentioned above shows that $S_n[E_z]$ and $C_n[H_z]$ are *even* functions of the u_i's. Description of the small circle around h_b thus restores an unmodified value of S_n and C_n, which shows that h_b is no branch point of the integrands in (27) and (28).

The only singularities of these integrands are the poles $\pm h_1$, $\pm h_2$, $\cdots$, $\pm h_n$, $\cdots$, which are zeros of Q and at the same time propagation constants of the modes. A finite number of them are on the real axis (propagated modes), the others on the imaginary axis (damped modes). Integration-segment $(-\infty, +\infty)$ in (27) and (28) can be replaced by a closed contour consisting of a half circle at infinity and the real axis. Both $S_n[E_z]$ and $C_n[H_z]$ have e^{ihz_0} as factor, so that the contour for $z > z_0$, for instance, must be taken, as in Fig. 2, to have a vanishing

integrand for the half circle. This contour does not include poles with negative real part, in accordance with Sommerfeld's radiation condition at infinity, the effect of which is to prevent waves coming from infinity toward the current-sources. If the integrals in (27) and (28) are replaced by the infinite sum of the residues at the poles, one finds that E_z and H_z are double expansions of the form

$$E_z = \sum_{n,k} A_{nk} \cdot e^{\pm i h_{nk}(z_0 - z)} \cdot f_{nk}(y) \cdot \sin \frac{n\pi x}{D} \cdot e^{i\omega t} \tag{29}$$

$$H_z = \sum_{n,k} B_{nk} \cdot e^{\pm i h_{nk}(z_0 - z)} \cdot g_{nk}(y) \cdot \cos \frac{n\pi x}{D} \cdot e^{i\omega t}. \tag{30}$$

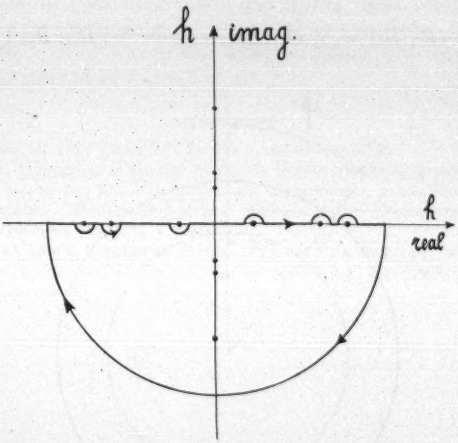


FIG. 2.

The + sign is to be taken for $z > z_0$, the − sign for $z < z_0$, and f_{nk}, g_{nk} are proportional to (19) to (24) where the y-dependence is concerned.

Full calculation of these expansions shows that

(a) coefficients A_{nk} and B_{nk} are equal.

(b) the expressions behind the coefficients A_{nk} and B_{nk} are, respectively, the E_z and H_z components of mode nk.

Furthermore, calculation of E_x and H_x from (6) and (7), together with E_y and H_y from similar expressions, also gives double expansions where the same coefficient A_{nk} multiplies the E_z, E_y, H_z, H_y components of mode nk.

In consequence, the whole Green's field $\bar{E}^z$, $\bar{H}^z$ due to current element (2) is expandable in normal modes, with coefficients A_{nk}, and each mode contributes waves propagated away from the current element.

Identical properties can be established in the same fashion for the fields $\bar{E}^y$, $\bar{H}^y$ and $\bar{E}^z$, $\bar{H}^z$ due to current elements in the y and z directions, respectively. The field excited by a current distribution $J(x_0y_0z_0)$ (J_x, J_y, J_z) is equivalent to a spatial distribution of current elements, because of the linear character of Maxwell's equations, and can therefore also be split into a sum of normal modes. To give more precision to this statement, let

$$(\bar{E}^x, \bar{H}^x) = \sum_{n,k} D_{nk}(x_0y_0) \cdot e^{\pm i h_{nk}(z_0 - z)} \cdot [\bar{E}_{nk}, \bar{H}_{nk}] \tag{31}$$

$$(\bar{E}^y, \bar{H}^y) = \sum_{n,k} F_{nk}(x_0y_0) \cdot e^{\pm i h_{nk}(z_0 - z)} \cdot [\bar{E}_{nk}, \bar{H}_{nk}] \tag{32}$$

$$(\bar{E}^z, \bar{H}^z) = \sum_{n,k} G_{nk}(x_0y_0) \cdot e^{\pm i h_{nk}(z_0 - z)} \cdot [\bar{E}_{nk}, \bar{H}_{nk}] \tag{33}$$

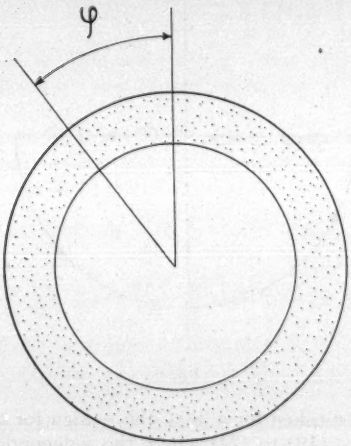


FIG. 3.

be the expansions of the Green's fields, where the brackets indicate the factor of the mode that depends on x and y. The field at some observation point (x, y) will be

$$(\bar{E}, \bar{H}) = \sum_{n,k} [\bar{E}_{nk}, \bar{H}_{nk}]$$
$$\times \Big[e^{-i h_{nk} z} \cdot \int_{J^+} (J_x \cdot D_{nk} + J_y \cdot F_{nk} + J_z \cdot G_{nk}) \cdot e^{i h_{nk} z_0} \cdot dx_0 dy_0 dz_0$$
$$+ e^{i h_{nk} z} \cdot \int_{J^-} (J_x \cdot D_{nk} + J_y \cdot F_{nk} + J_z \cdot G_{nk}) \cdot e^{-i h_{nk} z_0} \cdot dx_0 dy_0 dz_0 \Big]. \tag{34}$$

Volumes J^+ and J^- are those occupied by currents flowing at points $x_0y_0z_0$ whose z coordinate is, respectively, greater or smaller than that of the observation point. This formula shows that the expansion keeps constant coefficients between two cross-sections S_1 and S_2 which enclose no current, but that these coefficients vary as long as the cross-section of the observation point penetrates into current-carrying regions. This concludes the proof, the essence of which was to carry the expandability back to the simple scalar expandability (10) by the use of finite Fourier transforms.

An identical procedure can be applied to a rectangular guide containing several layers of various dielectrics, and to a circular guide containing several concentric dielectric layers, thus establishing the proof for the cross-sections most important in practice. In the latter case the finite Fourier transforms must be taken with respect to the φ's (Azimuth) instead of x (see Fig. 3).

REFERENCES

(1) J. VAN BLADEL, *J. Appl. Phys.*, Vol. 22, No. 1, pp. 68–69 (1951).
(2) R. B. ADLER, "Properties of Guided Waves on Inhomogeneous Cylindrical Structures," Technical Report No. 102, M.I.T. Res. Lab. of Electronics, p. 31 (1948).
(3) R. V. CHURCHILL, "Modern Operational Mathematics in Engineering," New York, McGraw-Hill Book Co., 1944, Chapter X.
(4) J. VAN BLADEL AND T. J. HIGGINS, *J. Appl. Phys.*, Vol. 22, No. 3, pp. 329–334 (1951).
(5) A. SOMMERFELD, "Partial Differential Equations in Physics," New York, Academic Press, Inc., 1949, p. 189.

Insecticidal Floor Wax.—A revolutionary household product that waxes floors and kills insects was introduced for the first time at the 47th annual convention of the National Food Brokers Association.

This new product, a combination high quality floor wax and powerful insecticide, is called Freewax, and is produced by the Freewax Corporation of Tallahassee, Florida.

Successfully pre-tested in thousands of Southern homes, it will be available to other consumers in the nation this Spring. Leading grocery stores, drug, hardware and department stores will sell Freewax for about 69¢ a pint.

"This new two-in-one product will make floors glow and bugs go," reports J. M. Feinberg, secretary-treasurer of the manufacturing firm. "It is the only product now in the market that offers homemakers, and other consumers, this dual service of shining floors and killing bugs in one operation.

"Freewax is a 'just apply-let dry' product that is self-polishing, and water resistant. It works on linoleum, asphalt and rubber tile, and cement as well as varnished and painted wood floors. It produces a high gloss without rubbing and tests have proved it to be skid resistant.

"The insecticide in the product is known as lindane. It is odorless and safe to human beings and pets. It kills roaches, ants, waterbugs, silverfish, and many other household insects that do millions of dollars worth of damage each year in American homes."

More than five years of research and 3000 laboratory tests conducted by Mr. Feinberg and his brother Irving, a chemist, led to the discovery and final development of Freewax, the world's first floor wax-insecticide.

During the past two years, market tests were successfully conducted in more than 100,000 homes in Florida. Tests by independent research laboratories disclosed that the new twofold product offers up to 100 per cent control of household crawling insects within 48 hr. after application.

Laboratory results also proved that Freewax meets the highest government specifications for floor waxes.

In addition to floors, Freewax can be used in other areas where insects run, such as cracks, crevices, baseboards, pantry shelves, around sinks, pipe openings, window and door sills. It can be applied with a paint brush or cloth.

Selenium Rectifiers.—International Resistance Company of Philadelphia has introduced its hermetically sealed Selenium Rectifiers with cell sizes as small as .060 in. diameter. Known as the Microstak line, these miniature units are available in a wide range of current and voltage ratings. The Hermetic Seal is made by metallized glass to metal bond which assures stable operation in high humidity, dust, corrosive fumes and changing pressures.

The design and small size of the Microstak rectifiers provide high frequency performance not found in conventional cells. These units have been used extensively in digital and analogue computers, hearing aids, air borne radar, CRT bias supplies and other advanced electronic circuits.

GENERALITIES CONCERNING "SIMPLE" AND MIXED PHOTOGRAPHIC EMULSIONS AS MANIFESTED BY THEIR CHARACTERISTIC CURVE. UNIQUENESS OF ANALYSIS. "INTERACTION" OF THE COMPONENTS OF A MIXTURE

BY

LUDWIK SILBERSTEIN[1]

The analysis of several hundreds of characteristic curves of various experimental (and some commercial) emulsions has shown that they can be very closely represented by either a single quantic term of the form

$$\alpha P_r(y) = \alpha \frac{y^r(r+1+y)}{(1+y)^{r+1}}, \qquad \alpha = \text{constant}, \; y = n\bar{a}\epsilon,^* \tag{1}$$

or by a binary combination of such terms, which (it will be recalled) are based on the schematic representation of the size-frequency distribution of the emulsion by the exponential function,

$$f(a) = e^{-a/\bar{a}} : \bar{a}.$$

We call an emulsion (or its characteristic curve) "simple" if its curve is well covered by a single term P_r and "mixed" if at least two such terms are necessary for a close representation of its curve, say

$$D(y) = \alpha P_{r_1}(y) + \beta P_{r_2}\left(\frac{y}{m}\right), \qquad \alpha + \beta = D_\infty, \tag{2}$$

where, generally, $m > 1$ and $r_1 \neq r_2$. Needless to say, if r_1 does differ from r_2, the expression (2) remains binary even if $m = 1$, that is to say, $\bar{a}_1\epsilon_1 = \bar{a}_2\epsilon_2$ (equal mean intrinsic sensitivities per grain); but if $r_1 = r_2$, the parameter m is supposed to differ from 1; otherwise the two component curves would coalesce into a single (simple) quantic curve.

Hitherto, in all our practice, binary combinations (such as in (2)) turned out to be sufficient.[2] For the sake of generality, however, we may contemplate a superposition of any number of quantic terms, that is,

$$D(y) = \Sigma\alpha P_r\left(\frac{y}{m}\right), \qquad \Sigma\alpha = D_\infty, \tag{3}$$

[1] Deceased; formerly, Research Consultant, Rochester, N. Y.

* The exposure y is equal to the product of the number, n, of incident quanta per square centimeter of emulsion layer surface, the average projective area of the grains, $\bar{a}$, and the intrinsic sensitivity of the grain, ϵ, which is the probability of the grain per unit area to become developable by the effective absorption of the minimum number of quanta, r.

[2] In a few cases, a third component quantic term, $P_r(y)$, seemed indispensable.

to be summed over all terms. For one of them the parameter m may, without loss to generality, be equated to 1, while it may have any other values in the remaining terms.

Keeping these simple remarks in mind, we may now make a substantial application of the main result of a preceding paper,[3] *viz.*, of the *uniqueness* of quantic representation of an observed curve, as proved.

Suppose, for instance, that an observed characteristic curve has been faithfully represented by the binary formula

$$D(y) = \alpha P_1(y) + \beta P_2\left(\frac{y}{m}\right), \qquad \alpha + \beta = D_m. \tag{3.2}$$

$P_1(y)$ is our previous $E(y)$ and $P_2(y/m)$ our previous $F(y/m)$.[4] The curve being thus represented as "mixed," *viz.*, as a superposition of a one-quantic and a two-quantic curve (the latter shifted with respect to the former rigidly through $\log m$ along the exposure axis), it is natural to infer that the emulsion showing this curve (with a given developer, at least) is itself *mixed*, that is, a *mixture* of, effectively, the amounts α/D_m and β/D_m of two component emulsions or two aggregates of grains, one calling for $r = 1$ and the other for $r = 2$ quanta as the minimum necessary for their developability and endowed with different intrinsic sensitivities ϵ_1, ϵ_2 and (possibly) different mean grain sizes, $\bar{a}_1$ and $\bar{a}_2$, such that, generally

$$\bar{a}_2\epsilon_2 = \frac{1}{m}\,\bar{a}_1\epsilon_1, \tag{4}$$

and, in particular, if there are weighty reasons for claiming $\bar{a}_1 = \bar{a}_2$, $\epsilon_2 = (1/m)\epsilon_1$.

This need not to be a deliberate mixture of two materials, say, prepared separately in two containers and then poured into a vessel and thoroughly stirred. The whole mass of emulsion might have been obtained by a single process and yet consist of two (or more) kinds of grains, differing in their acidity (r) as well as in their intrinsic sensitivity (ϵ) and mean grain size ($\bar{a}$).

The mathematical analysis of the given observed curve would thus disclose a mixed nature or constitution of the emulsion; a fraction α/D_m of one, and β/D_m of another "simple" emulsion characterized (in our present example) by $r = 1$, $\bar{a}_1$, ϵ_1 and $r = 2$, $\bar{a}_2\epsilon_2 = (1/m)\bar{a}_1\bar{\epsilon}_1$. Now, if the same observed curve could also be thoroughly represented by a binary, or higher, combination of other quantic terms, $\text{Const.} \times P_r(y/m)$, our analysis would be purely formal, not disclosing (that is) the actual compound structure of the emulsion.

[3] L. SILBERSTEIN, "Paucity of Quanta Rendering a Silver Halide Grain Developable. Uniqueness of Quantic Representation of a Simple Sensitometric Curve," JOUR. FRANKLIN INST., Vol. 251, p. 247 (1951).

[4] L. SILBERSTEIN, "Superposition or Mixture of Simple Quantic Characteristic Curves of Photographic Emulsions Layers. The Corresponding Relative Contrast," JOUR. FRANKLIN INST., Vol. 252, p. 309 (1951).

Owing, however, to the proof of *uniqueness* given previously,[3] this analysis does actually disclose the compound nature or the composition of the emulsion under investigation. And such knowledge is likely to be of genuine interest to the student of photography and especially to the emulsion maker.

To complete the discussion of this question, however, we must now generalize our proof. For we have, previously, considered[3] any number of terms P_r, but always with the same argument y, or $m = 1$ throughout, whereas in most actual cases the parameter m has turned out to have values considerably differing from (say, greater than) unity. It will suffice to consider here the case, (3.2), of a binary combination of quantic terms P_r.

Suppose, then, that the observed curve, being represented by formula (3.2), can also be covered by another such binary combination, with a partition ratio differing from $\alpha:\beta$ and with some other separation (m') of the two simple component curves, say by the combination

$$D(y) = \alpha' P_1(y) + \beta' P_2\left(\frac{y}{m'}\right), \qquad \alpha' + \beta' = D_\infty.$$

Now, recalling the meanings of the symbols P_1, P_2, we have the equation

$$\alpha \frac{y(2+y)}{(1+y)^2} + \beta \frac{y^2\left(3+\frac{y}{m}\right)}{m^2\left(1+\frac{y}{m}\right)^3} = \alpha' \frac{y(2+y)}{(1+y)^2} + \beta' \frac{y^2\left(3+\frac{y}{m'}\right)}{m'^2\left(1+\frac{y}{m'}\right)^3},$$

which is to hold for all exposures y. Hence, first of all, we may write

$$(\alpha - \alpha')\frac{2+y}{(1+y)^3} + \frac{\beta y(3m+y)}{(m+y)^3} - \frac{\beta' y(3m'+y)}{(m'+y)^3} = 0,$$

or, by Newton's binomial,

$$(\alpha - \alpha')(2+y)\{1 - 2y + 3y^2 - \cdots\}$$
$$+ y\left\{\frac{\beta}{m^3}(3m+y)\left(1 - 3\frac{y}{m} + 6\frac{y^2}{m^2} - \cdots\right)\right.$$
$$\left. - \frac{\beta'}{m'^3}(3m'+y)\left(1 - \frac{3y}{m'} + \frac{6y^2}{m'^2} - \cdots\right)\right\} = 0.$$

The coefficients of y^0, y, y^2, etc., must all vanish. Thus we have, first of all,

$$2(\alpha - \alpha') = 0, \text{ or } \alpha' = \alpha, \text{ and thus also } \beta' = \beta.$$

This leaves us with

$$\frac{1}{m^3}(3m+y)\left[1 - 3\frac{y}{m} + 6\frac{y^2}{m^2} - \cdots\right]$$
$$= \frac{1}{m'^3}(3m'+y)\left[1 - 3\frac{y}{m'} + 6\frac{y^2}{m'^2} - \cdots\right],$$

whence $3/m^2 = 3/m'^2$ or $m' = m$. In fine, $\alpha', \beta', m' = \alpha, \beta, m$, Q.E.D. The representation of the curve, and thus also the inferred constitution of the emulsion, is unique.

In much the same way the reader may deal with any other combinations of quantic terms, $P_r(y/m)$.

The *uniqueness* of representation of a given characteristic curve by a formula of the type (3), that is,

$$D(y) = \Sigma\alpha P_r\left(\frac{y}{m}\right), \qquad \Sigma\alpha = D_m,$$

is herewith completely proved. To restate it explicitly: If a given curve $D = D(y)$ is at all expressible (rigorously) by such a superposition of quantic terms, the number of these terms, and all parameters, α, r, m, are uniquely determined.

Practically, as we saw, a sufficiently close representation of curves observed on experimental emulsions calls only either for a single term, $P_r(y)$, or for a binary combination of such terms, as in (3.2).

The question of uniqueness of a representation by quantic formulas of the type specified above being thus settled, we shall now turn to consider a few particular topics which may be, theoretically at least, of some interest.

Although our very extensive trials have shown that quantum numbers (r) exceeding 3 are but sporadically required in representing a characteristic curve observed on an experimental emulsion, let us try to form an idea of the properties of a curve expressed by the formula

$$D(y) = D_m P_r(y) = D_m \frac{y^r(r + 1 + y)}{(1 + y)^{r+1}}$$

when r is, practically, a huge integer or, theoretically, when $r \to \infty$.

Whatever the exposure y, we may then, first of all, replace $r + 1$ in the numerator by r, and if we replace, for brevity, D/D_m by D, our formula may be written

$$D(y) = \frac{(r + y)y^r}{(1 + y)^{r+1}}. \tag{5}$$

As we know, the inflection point of the corresponding D, $\log y$ curve is given by $\bar{y} = r/2$. Let us consider the corresponding inflection density

$$\bar{D} = \frac{3}{2}\frac{r\cdot\left(\frac{r}{2}\right)^r}{\left(1 + \frac{r}{2}\right)^{r+1}} = 3\left(\frac{r}{2 + r}\right)^{r+1} = 3\left(1 + \frac{2}{r}\right)^{-r}.$$

This tends, for $r \to \infty$, to

$$\bar{D} = 3e^{-2r}. \tag{6}$$

Even for $r = 10$ only, we have $\bar{D} = 6.18\cdot 10^{-9}$, a practically evanescent

fraction of D_m. The limiting value of gamma, for $r \to \infty$, has none the less a finite, not evanescent, value, to wit, as we saw before,[5]

$$\gamma_{r=\infty} = \frac{4}{Me^2} \doteq 1.25.$$

Thus, in spite of the almost evanescent ordinate $\bar{D}$ at the inflection point, the D, $\log y$ curve is at that point rather steep, but not precipitously so.

For the *toe* of the curve, where y is negligible in the presence of r, we have

$$D = \frac{ry^r}{(1+y)^r} \to ry^r e^{-ry},$$

where y is a small fraction and r, an enormous number. The gradient of the curve,

$$g = \frac{y}{M} D'(y), \text{ is here}$$

$$g = \frac{1}{M} \frac{r^2 y^2}{(1+y)^r} = \frac{1}{M} r^2 y^r e^{-ry}.$$

If, for example, $r = 10$ and $y = \frac{1}{10}$, we have $g = \frac{1}{Me} 10^{-8} \doteq 0.85 \cdot 10^{-8}$, a practically evanescent gradient. With the same r-value, but $y = 1 = \frac{1}{5}\bar{y}$, the gradient is $g = \frac{100}{M} e^{-10} = 0.011$.

Since the inflection point is placed at $y = \bar{y} = r/2$, what is commonly referred to as *the shoulder* of the curve will correspond to exposures $y > r$. For $y \gg r$, we have

$$D = \left(\frac{y}{y+1}\right)^{r+1} \to e^{-\frac{r+1}{y}},$$

which, for $y \to \infty$, tends to unity (that is, to D_m).

The last formula holds, also, for *any* value of the quantum number r, amounting even to a few units of just 1, as is actually the case for most emulsions hitherto analyzed. In fact, the complete formula,

$$D = D_m \frac{y^r(r+1+y)}{(1+y)^{r+1}}$$

reduces, for all exposures, y considerably greater than $r+1$ (whatever

[5] L. SILBERSTEIN, "Macroscopic Sensitometry, Opacity and Photographic Density, Specular and Diffuse. The Quantic Exposure Law for a Uniform Emulsion," JOUR. FRANKLIN INST., Vol. 251, p. 359 (1951).

the value of r) to

$$\frac{D}{D_m} = \left(\frac{y}{1+y}\right)^{r+1} = \left(1+\frac{1}{y}\right)^{-(r+1)} = \left[\left(1+\frac{1}{y}\right)^{-y}\right]^{\frac{r+1}{y}},$$

and since, for $y \to \infty$, the bracketed expression tends to e^{-1}, we have

$$\frac{D}{D_m} = e^{-\frac{r+1}{y}} = 1 - \frac{r+1}{y} + \tfrac{1}{2}\frac{(r+1)^2}{y^2} - \cdots. \tag{7}$$

Up to the higher powers of the small term $\frac{r+1}{y}$,

$$\frac{D}{D_m} = 1 - \frac{r+1}{y},$$

for example, for $r = 1, 2, 3,$

$$1 - \frac{D}{D_m} = \frac{2}{y}, \frac{3}{y}, \frac{4}{y}.$$

The greater the quantum number r, the more sluggishly does the shoulder of the curve D, log y tend to the limiting density D_m.

Our second topic to be considered here concerns the sensitometric properties of *mixed* emulsions, namely, such emulsions whose components have been prepared separately and then deliberately mixed (stirred) in a common container. This subject, although wholly independent of the quantum theory (or, in fact, of *any* theory) of exposure, may be of interest and has, therefore, seemed to deserve some attention here.

Suppose, first, that two different emulsions, 1 and 2, have been spread on two pieces of film of negligible opacity, that the films thus coated have been exposed and developed, and then piled upon each other. Let D_1 and D_2 be the densities of the separate films and D that of the resultant double film. Then, since the opacity of the latter is $O = O_1O_2$ and $D = \log O$,

$$D = D_1 + D_2, \tag{8}$$

the well-known additive property of photographic densities. On the other hand, let $D_{1,2}$ be the density obtained on the thorough mixture of the same amounts of 1 and of 2 as have been previously spread over the component films, the exposure and development (if possible) being the same as in the first case.

Then, if there were no *interaction*[6] between the two ingredients of the mixed emulsion, one would, of course, expect $D_{1,2}$ to be simply equal to D or $D_1 + D_2$ for all exposures. On trial, however, we have found on a few materials, kindly prepared and measured by the Kodak Research Laboratories, that $D_{1,2}$ differs perceptibly from $D = D_1 + D_2$.

[6] Physical or chemical, or both, and photographically relevant.

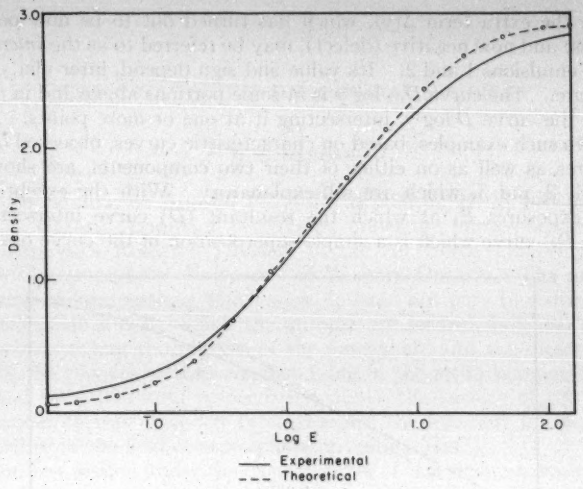


FIG. 1.

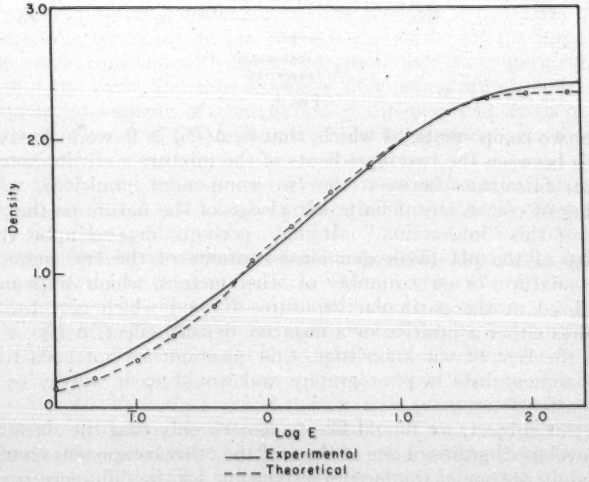


FIG. 2.

In symbols, $D_{1,2} = D_1 + D_2 + \Delta$ or, bringing into evidence the exposure y (varying from case to case),

$$D_{1,2}(y) = D_1(y) + D_2(y) + \Delta(y), \tag{9}$$

where the extra term $\Delta(y)$, which has turned out to be now positive (excess) and now negative (defect), may be referred to as the *interaction* of the emulsions 1 and 2. Its value and sign depend, inter alia, on the exposure. The curve $D_{1,2} \log y$ is in some portions above and in others below the curve $D \log y$, intersecting it at one or more points.

Two such examples, based on characteristic curves, observed on the mixtures as well as on either of their two components, are shown in Figs. 1, 2 and 3, which are self-explanatory. With the exception of these exposures E_0 at which the resultant (D) curve intersects the $(D_1 + D_2)$ curve which is a simple superposition of the curve observed

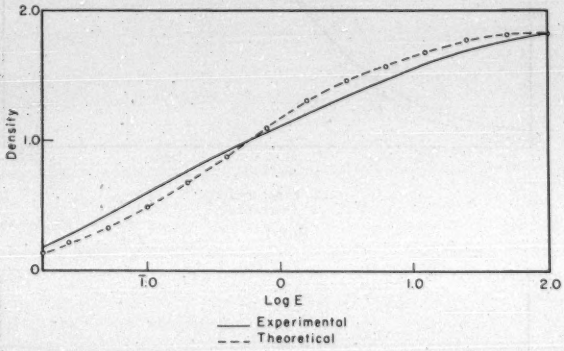


FIG. 3.

on the two components, at which, that is, $\Delta(E_0) = 0$, we may say that there is between the two ingredients of the mixture a sensitometrically relevant *interaction* between the two component emulsions, without claiming of course any definite knowledge of the nature or the mechanism of this "interaction." It may, perhaps, depend upon the inequality of the pH (hydrogen ions) contents of the two ingredients of the mixture, or on a number of other factors, which are somehow neutralized at the particular exposures E_0 and which give for other exposures either a positive or a negative density effect, $\Delta(E)$.

To the best of our knowledge, this question has not been treated by experimentalists in photography and would seem worthy of their active attention.

In this subject, we should like to remark only that the *shielding* of the (developed) grains of one of those of the other component emulsions is certainly *not* one of the factors responsible for the difference between D and $D_1 + D_2$. For any such shielding[5] has already been taken into account in deriving for the density or the Briggsian logarithm of the opacity the expression $0.4343\, k\bar{a}$, where $\bar{a}$ is the mean size (projective area) of a grain and k the *total* number of developed grains over unit area of the film or plate.

THE COLLOQUIUM ON AURORAL PHYSICS *

BY

N. C. GERSON[1]

A Colloquium on Auroral Physics was held on 27–28 July 1951 at London, Ontario, Dominion of Canada, under the joint sponsorship of the Geophysics Research Division of the Air Force Cambridge Research Center and the University of Western Ontario. The meeting consisted of four sessions which were devoted not only to a discussion of the mechanisms by which the aurorae are formed, but also to discussions regarding the physics of the ionosphere and the mesosphere; that is, the physics of those regions lying in the altitude range 80–400 km. and 400–1000 km., respectively, above the surface of the earth. The assembly was attended by a group of the foremost international authorities in the field of atmospheric investigations.

The first session under the chairmanship of B. Friedman was opened with a discussion on the subject "Electromagnetics as Applied to Auroral Problems." This session encompassed that phase of the general auroral problem which considered: (*a*) the passage of a neutral stream of ionized matter from the sun towards the earth; (*b*) the interaction of this neutral plasma with the geomagnetic field to cause a ring current (and the early and late phases of magnetic storms); and (*c*) the subsequent movement of charges along the magnetic lines of force towards the terrestrial atmosphere eventuating in the observed auroral luminosities.

The theory of Chapman and Ferraro was reviewed. This theory (based on empirical fact) assumes that the fundamental cause of the aurora may be attributed to a uniformly ionized neutral stream of particles, originally accelerated so that they may leave the sun, impinging upon the earth's atmosphere. After leaving the sun, the particles move towards the earth with little or no acceleration. At the earth's orbit the diameter of the somewhat diverging stream becomes about 10^{12} cm. The high conductivity of the advancing plasma allows the application of image theory to the problem; the total magnetic field at the earth's surface would therefore be enhanced by an amount equal to that produced by the image dipole. This rise in geomagnetic field strength constitutes the first phase of a magnetic storm. The subsequent formation of a "hollow" in the stream (because of its inter-

* This report was prepared especially for the JOURNAL. The Editors believe the information of sufficient general interest to warrant publication in the papers section.

[1] Geophysics Research Division, Air Force Cambridge Research Center, Cambridge, Mass.

action with the geomagnetic field) and the probable mechanisms giving rise to the main phase of a magnetic storm were discussed in some detail.

An idealized problem was considered; for example, a magnetic field with axial symmetry and a plasma approaching the axis radially from all directions. The forces arising from the interaction of the plasma with the magnetic field tend to move the positive ions and electrons in opposite directions. This movement of charges sets up a cylindrical current system in which the retarding force on the electrons is very much greater than that on the positive ions. Obviously from Gauss' theory, the field within the cylinder is zero. When the charge density is sufficiently small the difference in retarding forces separates the electrons and positive ions into two concentric cylindrical sheets. Solenoidal currents flow on the cylinders, the direction of the positive ion current being opposite to that of the electron current. For sufficiently high charge densities, the advancing sheet will be brought to rest before the opposite charges can separate.

If the conditions of the idealized problem are specialized so that the magnetic field corresponds to the earth's field at the magnetic equator and the plasma corresponds to that estimated for the solar corpuscular stream advancing towards the earth, it is found that the moving charges will be brought to rest at a distance of about fourteen earth radii from the axis and that the number of charges is about 10^{20} per unit length. (In the actual case of a dipole field and a stream approaching from one direction only, this stopping distance is about five earth radii.) The separation of the electron and positive ion cylinders when the ion density on the cylinder surface is 10^{2} per unit length would be about 2 cm.—a negligible quantity when considering the actual case.

Malmfors' experiment on the terrella was described. In this investigation a magnetized sphere was placed in a high electric field at low pressure so that the magnetic axis was perpendicular to the field. Luminosities were produced around the magnetic poles of the sphere both by electric breakdown of the surrounding gas and by shooting electrons into the container perpendicular to the electric field. An objection was raised that this and other similar experiments are unduly influenced by the walls of the container; thus the results may not be directly applicable to the aurora. Malmfors' work showed that the polar luminosities on the terrella were asymmetrical with respect to the magnetic axis. If his results may be extrapolated to the actual aurora the auroral rays should drift systematically to the west during darkness.

The question as to whether the particles bombarding the atmosphere to cause the aurora have equal velocities or equal energies during bombardment of the terrestrial atmosphere was raised. The influence of the solar magnetic field at the vicinity of the earth, and the possibility of a neutral ionized stream transporting a "frozen-in" magnetic field

were discussed. Although it applies more directly to a description of cosmic ray trajectories, the single charged particle theory of auroral formation was mentioned. In this theory a beam of charged particles (all of one sign) from the sun bombards the atmosphere to form the aurora; the field as well as the resulting great dispersion of such a beam at the distance of the earth's orbit from the sun is neglected. However, some convergence effects in this theory may assist the clarification of such phenomena as draperies, thin arcs, rays, etc.

Martyn's extension of the theories of Chapman and Ferraro on magnetic storms and aurorae was discussed.

A new theoretical approach utilizing a power series expansion in time of the electric and magnetic fields of the approaching plasma gave some insight into the initial transient phenomena. However, because of the nature of the series it can not be used for large values of time. Further, this time series may not bring out some aspects of the non-linearity of the problem.

It was concluded that further research was necessary at least on the following topics: (*a*) "frozen-in" magnetic fields transported by a neutral stream of ionized gas; (*b*) cylindrical streams of charged particles (of the same mass and charge) approaching a magnetic field (that is, an extension of Störmer's single charged particle theory to groups of particles arranged in cylindrical beams); and (*c*) the interaction of a plasma with an external magnetic field directed perpendicularly to the motion of the plasma.

It was generally agreed that much more effort was required on mathematical analyses describing the interaction of a neutral plasma with the geomagnetic field, eventually considering, for example, the displacement of the geomagnetic axis from the axis of rotation.

The second session, under the chairmanship of R. W. B. Pearse, considered the subject "Interpretation of Auroral and Airglow Spectra." The probable constituents in the ionosphere (80–400 km.) and mesosphere (400–1000 km.) as deduced from the auroral spectra were first examined. The molecular spectra possibly present in the aurora were taken to be: (*a*) the first negative system of ionized nitrogen; (*b*) the the Meinel bands of ionized nitrogen; (*c*) the first positive system of nitrogen; (*d*) the second positive system of nitrogen; (*e*) the Vegard-Kaplan bands of nitrogen; (*f*) the Goldstein-Kaplan bands of nitrogen; (*g*) the atmospheric bands of ionized oxygen; and (*h*) the Meinel bands of oxygen.

It was generally agreed that the first negative system of N_2^+ had been positively identified in auroral spectra leaving no doubt regarding its existence. A possible excitation mechanism was postulated as $N_2^{++} + e \rightarrow N_2^+ + h\nu$. The Meinel bands of N_2^+ were considered to be features of the aurora because of evidence presented regarding the doublet splitting, molecular constants and relative intensities in-

volved. The first and second positive systems of molecular nitrogen were also regarded as definite and reliable features of aurorae. The identifications of the Vegard-Kaplan bands in auroral spectra were deemed more questionable. One conferee stated that a single plate which he had analyzed indicated quite a few lines in a good position; otherwise, however, doubt was expressed as to whether the Vegard-Kaplan bands exist in the aurora. The presence of the Goldstein-Kaplan bands in the auroral spectrum was also doubted. It was agreed that the atmospheric bands of O_2^+ are present, particularly the (0, 1) transition; the (0, 0) transition is probably not observed because of absorption by the lower atmosphere. These bands are of about the same relative intensity but of stronger absolute intensity than those found in the night airglow. The Meinel oxygen bands are probably present.

The atomic lines possibly present in the auroral spectrum were taken to be those of hydrogen, oxygen, helium, nitrogen, and sodium. The lines of hydrogen may always be present and are strong in homogeneous arcs and flaming aurorae. The relative intensity of the hydrogen alpha line appears to be a maximum near the lower edge of the aurora, but decreases more rapidly with angular elevation than do some of the bands of ionized molecular nitrogen. It was suggested that auroral displays may be initiated by atomic hydrogen bombarding the mesosphere and ionosphere. There is no doubt regarding the presence of hydrogen lines.

It was agreed that the lines of OI were definitely present in the aurora. The probability that OII emissions occur, however, was viewed more cautiously; there are so many ionized levels that it is possible to find almost any desired coincidence. Reexamination of the lines now considered to arise from OII was deemed highly desirable. Although the evidence in favor of OII lines is becoming stronger, there was no general agreement at the present time that these lines exist in the auroral spectrum. It was considered doubtful that the lines of OIII, which would be found among the bands of N_2^+, are present in the auroral spectrum.

Considerable doubt was expressed regarding the presence of the lines of NI in the auroral spectrum. Some identification was claimed for the lines of NII, but it was suggested that attempts at confirmation be postponed pending reexamination of the auroral spectrum (for these lines) using a Fabry-Perot interferometer. It was agreed that there is no evidence to support the existence of NIII.

The lines of HeI and HeII were claimed to have been found in at least one auroral display. These lines may be natural features of the auroral spectrum; however, although the helium lines are possible, their presence in the auroral spectrum was not considered conclusive. It was suggested that the emissions of helium near 3188 Å (HeI) and

3203 Å (HeII) be sought, although they are probably hidden by the emissions from the second positive system of molecular nitrogen. The sodium doublet was not considered to arise in the aurora, but rather in the airglow from which the emission intrudes upon the auroral spectrum.

The probable molecular emissions of the airglow arise from the hydroxyl radical, the Vegard-Kaplan bands of nitrogen and the Herzberg and Meinel bands of oxygen. The OH lines are strong features of the night airglow. Ultraviolet bands of OH, if present, are probably masked by the Herzberg bands of O_2.

If recent work on band blending is accepted, the Vegard-Kaplan bands of N_2 appear rather weakly in the night airglow. Unfortunately, there are no calculated intensities with which to compare the results. Temperatures deduced from the V–K bands in the blue region of the spectrum were considered inaccurate because of the presence of the background continuum. In general, however, the lines are weak, and the probability of chance coincidences are high. More investigations were believed necessary before deciding whether or not the V–K bands are present in the night airglow. In the twilight airglow the first positive system of N_2 is present. (If the first positive system is not present, it is probable that the Vegard-Kaplan bands are absent.)

With regard to molecular oxygen, the Herzberg and Meinel systems are probably present in the night airglow.

The atomic lines which are found in the airglow are those of oxygen and sodium. In both cases the observations unmistakably disclose the lines of OI (the forbidden line being a definite and distinctive feature) and the doublet of sodium. Lines of NI might possibly be observable.

The third session, "Quantum Mechanics as Applied to Auroral Problems," was moderated by Ta-You-Wu. This meeting considered the vitally important but very much neglected field of cross sections for collision of atomic, molecular, and ionized particles found in the ionosphere and mesosphere (*a*) to each other and (*b*) to the auroral bombarding particles. Some comments were also made regarding the interaction of the atmospheric particles with photons emitted by the photosphere and the solar corona. In the case of photoionization and the reverse process, radiative positive ion-electron recombination, the coefficient is desired as a function of energy. The techniques of statistical mechanics should be employed in studying three-body and other problems. Similar considerations apply to photodissociation and its reverse process, association of the daughter particles to form the parent molecule.

In the case of the aurora and the airglow, the problem of electronic excitation and ionization by electrons requires much more examination. The main problems lie in the following three categories: (*a*) excitation by

slow electrons (the reverse process being deexcitation or quenching); (*b*) ionization by slow electrons (with various recombination processes including a possible three-body recombination involving two electrons); and (*c*) dissociation of molecules by slow electrons (the reverse process being three-body association). The exchange of electronic energy between two colliding particles, including the case of dissociative collision of a molecule when colliding with an excited atom, was also considered. Reverse processes are three-body association (when dissociation occurs) or resonance.

Very little is known regarding three-body collisions and much more study is needed. Mention was made of the exchange during collision of two forms of energy, such as

(*a*) Exchange of electronic and translational energy of heavy particles. These reactions may be typified by:

$$A + B^t \rightarrow A^e + B$$

or

$$AC + B^t \rightarrow (AC)^e + B.$$

(*b*) Exchange of the electronic and vibrational energy of molecules

$$AC + B^e \rightarrow (AC)^v + B$$

(the reverse process being termed quenching).

(*c*) Exchange of vibrational energy, and translational energy of electrons

$$AB + e \rightarrow (AB)^v + e$$

$$AB + e \rightarrow A + B + e.$$

All of the exchange reactions may be of considerable importance when dealing with collisions which occur in the high atmosphere.

Although all wave functions except those for hydrogen must be derived by approximate methods, the calculation of the cross section for photoionization of atoms may be considered as a relatively simple problem. Difficulties arise in the evaluation when the positive and negative contribution of the integral are about equal; in such a case, because the difference between two large numbers is obtained, the accuracy of the results is rather poor. Thus, for example, the agreement between experiment and theory is not good for neon, and the calculated values for potassium are very much in error. The difficulties in calculating the cross section for photodetachment from the negative oxygen atom arise because the wave functions for the negative ion are less accurately known than those of the atom and also because normalization of the continuous wave functions is not readily accomplished.

Theoretical determinations of the cross section for photoionization from molecules are laborious and intricate. Unpublished theoretical

results for O_2 (calculated by the Mulliken method) indicate a cross section of about 10^{-17} cm.^{2}, whereas experiments yield a value of about 10^{-20} cm.^{2}. In general, evaluations of collisional cross sections for molecules are difficult; cancellation is a critical item, the selection rules are far more restrictive; and several integrals are involved. The problem of radiative association of two oxygen atoms was discussed. The necessity for further research on molecular collision problems was emphasized.

In molecular collisions, if the time of collision compared to the time for the internal motions is short, accuracies comparable to those obtained in the optical region may be realized; in this case perturbation methods can be applied. Perturbation methods may also be utilized if one of the colliding particles is an electron, providing it has an energy comparable to that of the electrons in the atom. When the energy of the colliding electron exceeds five times the excitation energy of the heavy particle, methods applicable to the high energy region may be employed; that is, the Born approximation. With low energy electrons, mathematical treatments of the cross section for collisions of the type mentioned above are difficult. The DWBO approximation assumes weak coupling but considers the average field acting on the colliding electron. In problems of elastic scattering of atoms, gas kinetics, and transfer of excitation, the WKB approximation is useful.

With respect to slow collisions involving heavy particles the classical point of view may be used in many cases. In the case of three-body collisions to allow the association of O_2, the cross section may exceed the gas kinetic value by a factor of 10^8; experiments in physical chemistry support a value of 10^{-32} to 10^{-33}/cm.^{6}, but calculation under reasonable assumptions yields a value of only 10^{-36}/cm.^{6} for oxygen.

The fourth session, "Laboratory Investigations of Mesospheric Processes" under the direction of D. R. Bates, concerned itself with current laboratory experiments designed to clarify the mechanisms and processes occurring in the terrestrial ionosphere and mesosphere. An interesting experiment, perhaps more related to solar than mesospheric physics, concerned turbulence in liquid mercury which had been placed in a strong external magnetic field. When a strong, axial magnetic field is applied to a cylindrical vessel containing mercury being stirred by a nonconducting circular disc, vortices develop on the surface of the mercury in two well-defined patterns. Whereas ordinary turbulence is of the three dimensional type, "magnetic turbulence" may be considered to be two dimensional.

Some experiments are under way to measure the cross section for photodetachment of electrons from O^+ and H^+. Although it is also desired to determine the cross section for attachment of electrons to atoms of hydrogen and oxygen, a study of attachment in iodine will first be made. In the well known study of recombination in helium at

a pressure of about a few cm. Hg, a pulse microwave discharge is employed. The spectrum of He is found during the discharge, but that of He_2 is present in the afterglow. In order to clarify the reactions which occur, this type of experiment will be continued and "everything possible" will be measured.

The experiment to measure very low pressure recombination (described at the Colloquium on Mesospheric Physics held at Cambridge, Massachusetts during July–August 1950)[2] is continuing. A toroidal discharge tube with an associated coannular magnetic field will be utilized in an attempt to minimize diffusion to the walls of the container. No results are yet available.

A comprehensive investigation is in progress to measure the electrical conditions in a discharge plasma, using seven probes in the discharge tube. The temperature and concentrations of both the electrons and ions will be measured and the relative intensities of the emitted band spectra determined. The experiment will be made using nitrogen at low pressure, and the spectral results will be compared with computations of the fractional transition probabilities.

Another experiment will attempt to measure (by coincidence counting techniques using photomultiplier tubes) rates of transition between different energy levels of nitrogen. From the observed coincidence rate as a function of delay time, the lifetime of the atom in the second energy level may be derived. Measurements are being made of cross sections for charge exchange between positive ions and neutral atoms. Work is also being undertaken to measure cross sections for transfer of excitation between normal and metastable atoms. Beam experiments, in which electrons and positive ions of various energies are shot into given gases and the resulting excitations determined, are also in progress.

It was mentioned that many processes involving the minor atmospheric constituents are possible but that the rate of the reactions involved is unknown. These processes involve the formation and dissociation of HO_2, N_2O, CO_2, OH, O_3, etc. With regard to the emission of the sodium doublet, the question was raised as to whether the mechanism

$$NaO + O \rightarrow Na + O_2$$

is energetically possible (as had been postulated) and if so, what the value of the reaction rate is.

The production of appreciable quantities of atomic hydrogen for use in the laboratory was mentioned.

The Colloquium was highlighted by the presentation of a gift to Carl Störmer by S. Chapman on behalf of the conferees. The gift was in honor of Störmer's distinguished career in the field of auroral physics, a field which he pioneered and in which he contributed fundamental papers of great importance.

[2] See this JOURNAL, Vol. 250, p. 472 (1950).

NOTES FROM THE FRANKLIN INSTITUTE LABORATORIES FOR RESEARCH AND DEVELOPMENT

A MONITOR DEVICE FOR RAILROAD TRACK RESISTANCE

Railroad passenger and freight cars, which are being constructed today, provide increased comfort and safety to riders and shippers. The importance of safety requirements points up the necessity for mutual cooperation between equipment suppliers and the operating railroads, because the latter provide the signaling systems that will ensure safe operation of rolling stock.

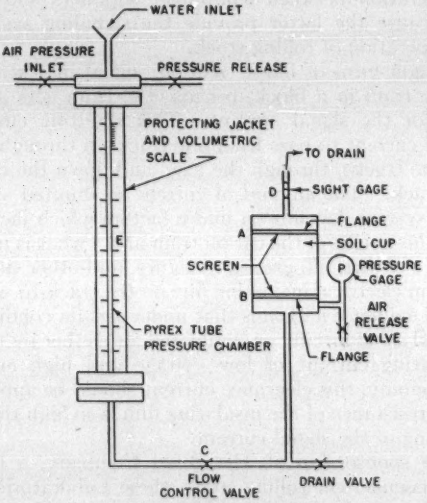
One common form of block (section) signal indicates the presence of a coach or train in a block, because the train acts as an electrical shunt path for the signal current. This shunting effect causes the signal system current to pass from one track up through the wheels on one side of the trucks, through the axle, and down the other wheels to the second track. The amount of current so shunted and the efficacy of the signal system depends on many factors which include the speed of the train, the weight of the car on train and—what is most important—the presence of dirt, oil, grease, moisture, and other similar materials which can form electrical insulating film on the track-to-wheel interface. Experimental evidence indicates that under certain conditions, this film can be reduced substantially or eliminated altogether by the application of an alternating current of low voltage and high amperage. For reasons of economy, this clearance current should be applied only when the electrical resistance of the insulating film is so high that it precludes proper shunting of the signal current.

Under the sponsorship of The Budd Company, a leading manufacturer of passenger-car rolling stock, these Laboratories developed a monitoring device and controller which measures the electrical resistance of the insulating film and initiates the application of a clearance current when required. The device is in principle a sequentially operating resistance bridge one arm of which is the wheel-to-track interface resistance. If a high-resistance film is encountered, the unbalance current of the bridge actuates a relay which applies the clearance current for a pre-set interval of time, at the end of which the monitoring process is repeated. Controls are provided to vary the time during which the clearance current is applied and to account for the different ambient track conditions encountered on different roadbeds, for example, moisture, oil, dirt, etc. Preliminary tests of this device on tracks of a major eastern railroad indicated its usefulness as a research tool and as a potential safety device for installation on railroad cars and trains.

S. CHARP

SOIL PERMEABILITY TESTER

The blending and treating of soils to increase their bearing strengths, reduce their permeabilities to water, and improve their resistance to erosion or caking is a fast-growing field for technological endeavor. Some important successful applications are: the stabilizing and strengthening of the sub-grades under pavements; strengthening the soil under foundations and structures; increasing the service-life of adobe bricks; shutting off the seepage of water around dams, swimming pools, tunnels, and basements; hardening the soil and speeding the drainage of barnyards.



Schematic diagram of the Soil Permeability Tester.

The soil permeability tester was designed and built by the Colloids Section of these Laboratories. The tester provides measurements of the rate of flow of water at various pressures, through treated and untreated soils. It is also used to measure roughly the void content or porosity of soils. Most soils contain 30 to 60 per cent by volume of air, and the underlying idea of many treatment processes is to reduce this porosity as much as possible.

In using the tester to measure porosity, the soil is packed into the cup, between screens *A* and *B*. Valve *C* is opened, water flows upward slowly through the soil to level *D*, and the volume of water required to

fill the apparatus is read on scale *E*. The test is repeated with the soil cup empty; from the difference in volumes, the volume of solids in the cup and the soil porosity are easily calculated.

To measure water permeability at various pressures, the soil is packed in the soil cup, and water is permitted to flow through it to level *D*. Now the pressure in chamber *F* is increased to the desired figure, using a regulated-pressure air supply, and the rate of flow of water through the soil is calculated from the rate at which the water level drops in chamber *F*. Water permeability often changes as the flow of water through the soil continues, and this change with total flow provides useful information regarding the permanence of the soil treatment and the structure of the treated soil.

EDMUND THELEN

NOTES FROM THE NATIONAL BUREAU OF STANDARDS *

NBS TELEMETERING IN-FLIGHT CALIBRATOR

A 10-channel in-flight calibrator recently developed by the National Bureau of Standards utilizes a unique system of cam-operated switches that eliminates many of the difficulties encountered in calibrators of comparable size and scope. Designed to identify telemetered information from a guided missile in flight, the device is compact and durable and particularly suited to telemetering applications in aircraft, missiles, or projectiles. The initial model, which was designed by L. L. Parker and W. A. Hereth of the NBS guided missiles laboratory, is now being manufactured by the J. P. Seeburg Company.

Telemetering involves the measurement of one or more quantities by electrical instruments, the transmission of the data to a distant receiving station, and the receipt and recording of the measured quantities. The information is usually transmitted by radio, although under some circumstances it may be carried by wire. The transmitted signals may be derived from transducers or pickups which translate mechanical movement into electrical impulses or they may be derived through direct coupling to electrical circuits. Normally, the information is then used to modulate a high-frequency radio carrier.

At the receiver-recorder it is virtually impossible to assign any finite value to the received information until two or more reference levels of modulation have been established in terms of the response of the receiving equipment. The NBS in-flight calibrator sequentially supplies each intelligence channel with four known levels of modulation. Thus, the telemetering record is provided with known reference levels of modulation from which the received data may be interpreted.

Transmission control of the calibrating signal, as well as the "data" signal, is achieved through the action of two groups of noise-free switches involving 40 separate switching operations. The first group contains 10 single-pole double-throw switches operated in sequence by a motor-driven cam. Its function is to interrupt briefly and periodically the normal channeling of the operating information to the r-f. transmitter and to connect the calibrating circuit to the output, instead.

The second switching-group contains four single-pole single-throw switches, also operated by a motor-driven cam. The function of this group is to pass step-calibrating signals sequentially to the transmitter during the interval in which a particular telemetering channel is interrupted by the first group of switches. The step-calibrating cam cycles 10 times (four modulating signals with each cycle) for each cycle of the channel-interrupting cam; each interrupted channel receives identical calibrating signals.

* Communicated by the Director.

All switches and cams are contained within a cylinder about 2 inches long and 4 inches in diameter. The drive motor for both cams is mounted at one end of the calibrator.

The NBS in-flight calibrator could easily be expanded to permit operation of 20 telemetering channels. This would simply involve installing 10 more switches, increasing the width of the slow-speed cam, and enlarging the calibrator housing to accommodate the additional switches. Such an arrangement would permit two channels to be calibrated simultaneously; each channel would be recalibrated every 10 calibration cycles.

The four reference calibration levels supplied by the NBS calibrator to the transmitter are usually chosen so as to divide the total modulation range into equal parts on the record. Thus, the traces of the calibration signals not only form a basis for assigning calibration values to the record but also provide a means for correcting for any non-linear qualities of the transmitter and receiver-recorder.

Other useful functions are served by the calibrator. In some recording systems it is desirable to increase the sensitivity of the telemetering receiving equipment to such a degree that adjacent recording traces overlap; the sequential nature of the NBS calibrator then serves to identify the separate traces, which may become confused because of the many cross-over points. The mechanism is also useful in applying modulation to the telemetering transmitter during pre-flight adjustment and checkout. In effect, the calibrator simulates the information normally translated by the transducers and thus permits regulation and testing of the transmitter-receiver system prior to actual flight.

The NBS in-flight calibrator performs satisfactorily under a wide range of drive-motor speeds; however, the optimum speed for in-flight operation of the high-speed cam is around 200 rpm. To prolong the life of the components, it is desirable during pre-flight adjustment to operate the in-flight calibrator at a lower speed of about 60 rpm.

SOURCE OF DEFECT-PRODUCING HYDROGEN IN PORCELAIN-ENAMELED STEEL

Hydrogen gas evolved during and after the firing operation has for some time been recognized as a cause of defects in porcelain-enameled and ceramic-coated steel. At high temperatures the gas may cause blistering during the firing, or "reboil" (the blistering of a vitreous coating when reheated after the first firing). At ordinary temperatures, if the gas is trapped in the steel during rapid cooling after firing and later escapes, "fishscaling" may occur; gas pressure opens scale-like fissures, and in some cases literally blows off flakes of the coating. Until now, the relative importance of the different possible sources of the hydrogen causing the defects has been largely a matter of conjecture. Uncertainty as to the origin of the gas has made the problem difficult to cope with effectively.

A recent National Bureau of Standards investigation, using heavy hydrogen as a tracer, now indicates that water present in the frit (glass) used in the coating is largely to blame for hydrogen-produced defects. Moreover, confirming experiments demonstrate the feasibility of producing a coating free from hydrogen defects, by using a water-free frit. Results of the new NBS study, which was undertaken for the National Advisory Committee for Aeronautics by D. G. Moore, Mary A. Mason, and W. N. Harrison of the NBS staff, should enable manufacturers to produce better porcelain-enameled steel.

At the beginning of the investigation it was considered that defect-producing hydrogen could conceivably originate in any of a number of ways. First, there is the acid pickling operation used to clean the steel; some of the hydrogen formed by the acid-steel reaction might be occluded by the metal. Second, any water present during the firing operation could react with the iron in the steel to form iron oxide and hydrogen. Possible sources of water during firing appeared to be (*a*) water vapor in the furnace; (*b*) the water used for milling and suspending the enamel slip; (*c*) the water chemically combined in the clay used for suspending the slip; or (*d*) dissolved water in the frit used in the enamel.

The NBS investigation sought to establish the relative importance of these five possible sources. The problem was attacked by substituting heavy hydrogen (deuterium), in the form of heavy water, for ordinary water at each of these five possible hydrogen-producing points in the coating process. Heavy hydrogen and ordinary hydrogen have almost identical chemical properties and very similar physical properties. But because they differ in mass, the two isotopes can be readily separated with the mass spectrometer.

In a series of experiments the heavy-hydrogen substitution was made at one point at a time. Different techniques were required to effect the desired substitution at the different points. In the case of the dissolved water in the frit, the substitution involved heating normally smelted frit under reduced pressure to remove the ordinary water, then heating with heavy water under increased pressure.

In each group of experiments samples of two steels were coated: 18-gage enameling iron and a 10-gage low-carbon steel that had given considerable trouble from delayed fishscaling in one enameling plant—a coating showing poor adherence was used so that defects would be accentuated. The gases given off by each enameled-steel specimen after firing were collected for analysis. This was done by quickly placing the fired specimen in a glass collection cell and heating to 338° F. at reduced pressure for 17 hr. The cell was then allowed to cool and the pressure noted.

The ratio of heavy hydrogen to total hydrogen in the collection cell, as shown by the mass spectrometer, varied widely with the particular source under study (Table I). When the frit had heavy water substi-

TABLE I.—*Volume of Hydrogen Gas Collected, Mole Per Cent Deuterium in the Hydrogen, and Relative Importance of the Investigated Sources of Defect-Producing Hydrogen in Steel*

Source Investigated	Steel Base	Volume of[1] Hydrogen Collected cm.³/100 g. steel	Mole Per Cent D_2 in Hydrogen	Relative Importance of Source
Pickling acid	A	0.27	0.1	4
	B		0.5	
Milling water	A	0.24	0.1	
	B	0.32	0.1	5
Water in clay	A	0.25	0.2	
	B	0.28	0.8	3
Quenching water	A	0.80	0.5	
	B	0.20	1.5	2
Dissolved water in frit	A	0.56	53.3	
	B	0.44	52.9	1

[1] Computed from change in pressure in collection cell after 17 hr. at 338° F. and from the hydrogen content of collected gas. Values are for gas at standard conditions.

tuted for ordinary water, heavy hydrogen constituted about 53 mole per cent of total hydrogen recovered. In contrast, the corresponding figure was only about 1 per cent when what proved to be the next most important source of hydrogen—the quenching water—was under study. The acid pickling process, the milling water, the chemically combined water in the clay, and the quenching water thus all proved to be minor sources of defect-producing hydrogen under the experimental conditions. The 18-gage enameling iron and the 10-gage low-carbon steel behaved similarly in these experiments.

The finding with the heavy hydrogen tracer—that dissolved water in the frit was the principal source of defect-producing hydrogen—was confirmed by supplemental experiments. In these, steel specimens coated with normally prepared frit were compared for defects with specimens coated with frit from which dissolved water had been largely or entirely removed. In each case the frit was applied without the use of the customary clay, water, and other mill additions. This was done by dry-grinding the frit, suspending it in a water-free organic solvent, and spraying it on. After air-drying, the specimens were fired for the normal time.

Two methods of preparing water-free frit were used in these experiments: drying normally prepared frit by vacuum melting, and preparing frit from dried raw materials. Neither lifting nor fishscaling occurred with the vacuum-melted frit, although both these defects were pronounced with the normally prepared frit. Other experiments with reboiling gave confirmatory results. Attempts to prepare defect-free frit from raw materials from which most of the water had been removed were somewhat less successful; some defects developed, though not nearly as many as with the normally prepared frit.

THE FRANKLIN INSTITUTE

MINUTES OF THE STATED MEETING
March 19, 1952

The Stated Monthly Meeting of The Franklin Institute was held on March 19, 1952, in the Lecture Hall of The Franklin Institute. S. Wyman Rolph, President, called the meeting to order at 8:20 P.M. Approximately 120 persons attended.

The President stated that the Minutes of the Annual Meeting in January were printed in full in the February issue of the JOURNAL; there being no corrections or additions, they were approved as printed. The Minutes of the Stated Monthly Meeting in February, which are being printed in full in the March issue of the JOURNAL, will be submitted for approval at the April meeting.

The Secretary reported the following elections to membership for the month of February:

Active	36
Associate	29
Student	19

and a total Institute membership of 6,283 as of February 29, 1952.

He announced that the National Asphalt Research Center, a project of The Franklin Institute Laboratories for Research and Development, held its second annual meeting at the Institute on February 28th and 29th. Fifty-four representatives of the twenty-six sponsors attended the meeting. The N.A.R.C. is one of the many projects of the Institute's industrial research laboratories, which now employ 284 persons.

The Secretary also called attention to the recently-revamped Illusions Exhibit in the Museum where there are many examples of things not being what they appear to be.

The President then welcomed the officers and members of the Philadelphia Society for Promoting Agriculture to its 12th Joint Meeting with the Institute. After speaking briefly of the Society, Mr. Rolph presented its President, Mr. J. Gordon Fetterman, who took the chair. Mr. Fetterman spoke of the pleasant relationship the Society has enjoyed with the Institute and introduced the speaker for the evening—Dr. Firman E. Bear, Chairman of the Department of Soils, Rutgers University.

In his lecture on "The Earth, and the Fullness Thereof," Dr. Bear spoke of soil conditions in the United States from the time when the population consisted of 800,000 Indians to the present time when the population totals 156,000,000. He also spoke of what conditions would be like if the population ever reaches the 2,000,000,000 mark. The lecture was both interesting and stimulating, as was evidenced by the lively question-and-answer period that followed.

The meeting adjourned at 9:55 P.M. with a rising vote of thanks to Dr. Bear.

HENRY B. ALLEN,
Secretary

COMMITTEE ON SCIENCE AND THE ARTS

(Abstract of Proceedings of Stated Meeting held Wednesday, March 12, 1952.)

HALL OF THE COMMITTEE,
PHILADELPHIA, MARCH 12, 1952.

MR. HOWARD STOERTZ in the Chair.

The following reports were presented for final action:

No. 3237: "Pyrex Vycor" High Silica Laboratory Glassware.

This report recommended the award of a John Price Wetherill Medal, each, to Harrison P. Hood and Martin E. Nordberg, both of Corning, New York, "In consideration of the discovery of a truly novel process for the manufacture of unusual, high silica glasses comprising the vitrification of an insoluble phase of a heat-treated alkali-borosilicate glass after its soluble phase has been leached with inorganic acid."

No. 3247: Ballantine Medal.

This report recommended the award of a Ballantine Medal, each, to John Bardeen, of Urbana, Illinois, and Walter H. Brattain, of Morristown, New Jersey, "In recognition of their contributions to the theory of surface conduction in semi-conductors and of their consequent invention of the Point Contact Transistor, a device foreshadowing a notable advance in the means of electromagnetic communications."

JOHN FRAZER,
Secretary to Committee

MEMBERSHIP

ACTIVE MEMBERS ELECTED AT THE MEETING OF THE BOARD OF MANAGERS, MARCH 19, 1952

ACTIVE FAMILY

Allen G. King	Harry S. Nitterauer	Paul Weizer

ACTIVE

Donald M. Blair	Henry H. Gildner	John A. Nigro, Jr.
Earl F. Blinn	Fred Green, Sr.	Daniel Scardino
Nathaniel Brewer	Herbert S. Henderson	Clifford E. Sifton, Jr.
Louis A. Brooks	Wallaston K. James	Harry F. Stern, II
Albert C. Burger	William Johnson, Jr.	Frederick A. Stuart
Nathan Cohen	Louis Luberoff	John S. Walton
George F. Deck	Frederick C. McCormack, Jr.	Richard G. Weston
Alfred Dill	Isidor Moskowitz	J. William Wills

ACTIVE NON-RESIDENT

Donald E. Dawson	Henry S. Evans	Charles Roger Sutton
	Warren Josephs	

NECROLOGY

Mrs. Arthur Biddle '36	Alfred G. Heineman '43	Mrs. Charles Mortimoore '36
Alfred W. Charlton '36	Harry E. Kohn '36	J. Jacob Schoening, M.D. '46
Paul C. Girard '51		C. Frederick Stout '36

LIBRARY

The Committee on Library desires to add to the collections any technical works that members may wish to contribute. Contributions will be gratefully acknowledged and placed in the library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat Service. Photostat prints of any material in the collections can be supplied on request.

The Library and reading room are open on Mondays, Tuesdays, Fridays and Saturdays from 9 A.M. until 5 P.M.; Wednesdays and Thursdays from 2 P.M. until 10 P.M.

RECENT ADDITIONS

AERODYNAMICS

DURHAM, FRANKLIN P. Aircraft Jet Powerplants. 1951.

ARCHITECTURE AND BUILDING

BRÜEL, PER V. Sound Insulation and Room Acoustics. 1951.

ASTRONOMY

HARANG, L. The Aurorae. 1951.

BIOGRAPHY

DUNHAM, AUSTIN C. Reminiscences of Austin C. Dunham. n.d.
THOMPSON, SILVANUS P. Michael Faraday; His Life and Work. 1898.

BIOLOGICAL CHEMISTRY

MARTIN, GUSTAV J. Biological Antagonism. 1951.

CHEMISTRY AND CHEMICAL TECHNOLOGY

Advances in Carbohydrate Chemistry. Vol. 6. 1951.
BRANNT, WILLIAM T. AND WAHL, W. H., comp. Techno Chemical Receipt Book. 1888.
DEUEL, HARRY JAMES, JR. The Lipids. Vol. 1. 1951.
Encyclopedia of Hydrocarbon Compounds. Vol. 10. 1951.
FETKENHEUER, BRUNO. Ueber Ammoniakverbindungen der Halogenide. 1914.
GROVES, A. W. Silicate Analysis. Ed. 2. 1951.
HASZELDINE, R. N. AND SHARPE, A. G. Fluorine and Its Compounds. 1951.
NORST, ELSE. Über die Kernstruktur der Atome. 1921.
PORTER, M. W. AND SPILLER, R. C. Barker Index of Crystals. Vol. 1–2. 1951.
RICHTER, V[ICTOR VON]. Chemie der Kohlenstoffrerbind Ungen. Ed. 11. Vol. 1 & 2. 1909–13.
RODD, E. H., ed. Chemistry of Carbon Coumpounds. Vol. 1. 1951.
STOCK, ALFRED. Ultra Strukturchemie. 1920.
UTERMARK, WALTHER. Schmelzpunkt Tabellen Organischer Verbindungen. 1951.

CIVIL ENGINEERING

BLEICH, FRIEDRICH. Buckling Strength of Metal Structures. 1952.
SEELYE, ELWYN E. Data Book for Civil Engineers. Ed. 2. 1951.

DICTIONARIES

CAMPBELL, F. H. Chemical Dictionary. 1941.

ELECTRICITY AND ELECTRICAL ENGINEERING

BADT, F[RANCIS] B[EATUS]. Electric Transmission Handbook. 1891.
BERG, ERNST JULIUS. Electrical Energy. 1908.
GENERAL ELECTRIC CO. Instructions for Testing Electrical Apparatus. c1909.
KLUTH, HEINRICH. Jeder Sein Eigener Schallplattenfabrikant. 1932.
LE PAGE, WILBUR REED AND SEELY, SAMUEL. General Network Analysis. 1952.

ELECTRONICS

DORF, RICHARD H. AND KAMEN, IRA. TV and Electronics as a Career. [1951.]
FINK, DONALD G. Television Engineering. Ed. 2. 1952.
SONNENBERG, G. L. Radar and Electronic Navigation. 1951.

ENGINEERING

BRIDGMAN, PERCY WILLIAM. Studies in Large Plastic Flow and Fracture. 1952.
MURNAGHAN, FRANCIS D. Finite Deformation of an Elastic Solid. 1951.

MANUFACTURE

BIDLACK, VERNE C. AND FASIG, EDGAR W., ED. Paint and Varnish Production Manual. 1951.

MATHEMATICS

ABBOTT, W. Practical Geometry and Engineering Graphics. Ed. 5. 1951.
BRIDGMAN, P. W. Dimensional Analysis. 1922.
GILLESPIE, R. P. Partial Differentiation. 1951.
HAMBURGER, H. L. AND GRIMSHAW, M. E. Linear Transformations in N-Dimensional Vector Space. 1951.
HARVARD UNIVERSITY. COMPUTATION LABORATORY. Tables of the Error Function. 1952.
TODHUNTER, E. A Treatise on the Integral Calculus. 1886.
VILLARS, DONALD STATLER. Statistical Design and Analysis of Experiments for Development Research. 1951.

MECHANICAL ENGINEERING

SIMONS, ERIC N. Cutting-Tool Materials. 1951.

METALLURGY

GUY, ALBERT G. Elements of Physical Metallurgy. 1951.

OPTICS

BREWSTER, SIR DAVID. A Treatise on Optics. [1853.]
TAYLOR, DENNIS. A System of Applied Optics. 1906.

PHOTOGRAPHY

ANDRESEN, M. Das Latente Lichtbild. 1913.
DUDLEY, LESLIE P. Stereoptics. 1951.
LÜPPO-CRAMER. Die Trockenplatte. n.d.
MEYER, HUGO AND CO. Hugo-Meyer-Photo-Optik. Vol. 4. 1920.
MIETHE, A. Dreifarbenphotographie nach der Natur. Ed. 2. 1908.
Photograms of the Year 1952. 1951.

PHYSICS

CONDON, EDWARD U. AND SHORTLEY, GEORGE H. The Theory of Atomic Spectra. 1951.
FECHNER, GUSTAV THEODOR. Elementar-Lehrbuch des Elektromagnetismas. 1830.
HENRY, N. F. M., LIPSON, H. AND WOOSTER, WILLIAM A. The Interpretation of X-ray Diffraction Photographs. 1951.
MERIAM, J. L. Mechanics. Pt. 1. 1952.
POWELL, RALPH WATERBURY. An Elementary Text in Hydraulics and Fluid Mechanics. [1951.]

PLASTICS

DELMONTE, JOHN. Plastic Molding. 1952.
LEARMONTH, G. S. Laminated Plastics. 1951.

RADIO

SPREEN, WILHELM. Die Physikalischen Grundlagen der Radiotechnik. 1924.

SCIENCE

DAVY, SIR HUMPHREY. Six Discourses Delivered before the Royal Society. 1827.

TEXTILES

LÜDKE, WERNER. Die Empfindlichkeit der Kunstfasern Gegen Wasser ein Beitrag zu ihrer Erkenntnis. 1928.

READ, JOHN. Elementary Textile Design and Fabric Structure. 1950.

JOURNAL OF THE FRANKLIN INSTITUTE

The following papers will appear in the JOURNAL within the next few months:

MOON, PARRY AND DOMINA EBERLE SPENCER: Separability Conditions for the Laplace and Helmholtz Equations.

BISPLINGHOFF, R. L. AND C. S. DOHERTY: Some Studies of the Impact of Vee Wedges on a Water Surface.

SAIBEL, EDWARD: Buckling of Continuous Beams on Elastic Supports.

RYDER, FREDERICK L.: Network Analysis by Least Power Theorems.

MINORSKY, N.: Stationary Solutions of Certain Nonlinear Differential Equations.

TOMOVICH, RAJKO: A Universal Unit for the Electrical Differential Analyser.

The May issue will be devoted entirely to the subject of "Lightning Protection," in honor of the 200th anniversary of the successful proof of Benjamin Franklin's theory that lightning, like electricity, could be attracted by points. Four papers by eminent authorities in the lightning protection field will be offered.

THE FRANKLIN INSTITUTE

exists today because of the faith and generosity of the men and women who for 128 years have given time and money to its support.

The Institute welcomes financial gifts and bequests and hopes that all those who desire to perpetuate its work will make the The Institute one of their beneficiaries.

FORM OF BEQUEST

*I give and bequeath to The Franklin Institute of the State of Pennsylvania for the Promotion of the Mechanic Arts, the sum of*_____

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION

ELLICE McDONALD, DIRECTOR

CANCER RESEARCH

BY

ELLICE McDONALD AND FRANCIS B. CRAMER

In any research effort the first task is to define the problem. In vital systems, this is difficult on account of the complex metabolism (energy adaptation) and the many unknown variables. But it is better to define the problem broadly and to let further experimentation refine the understanding. The most important and obvious manifestation of cancer is the rapid growth, reproduction or proliferation of cancer cells. But to state the problem thus is only begging the question: there remains the *mechanism* of growth and cell division which obviously requires the metabolism of energy-supplying metabolites through enzymatic pathways.

Chemotherapy in its broadest aspect is a reflection of substrate displacement and enzymatic inactivation. But to this time the search has been for enzymatic inactivators while the enzymatic processes involved were imperfectly understood. The Biochemical Research Foundation began its enzyme study in 1932 and published its first paper (1) on enzymes in cancer which did not show much except that there were a lot of enzymes associated with cancer. The search for cancer inhibitors has gone along the lines of a specific substance which will cure cancer, something of the nature of a miracle without explaining the mechanism of its action. This sort of a miracle is dear to human belief because it has seen so many such as quinine in malaria, mercury in syphilis, the vaccines and serums in infections, etc. It has made possible the innumerable "cures" for cancer which seize hold of men's minds and which require so much time to evaluate and disprove.

In all this the search has been for enzyme inhibitors, and in other fields than cancer there have been certain successes, as for example the competition offered by the sulpha drugs to attachment of para-aminobenzoic acid to an enzyme essential to the vital economy of certain pathogenic microorganisms. The result has been a new concept, the theory of biological antagonism dependent upon structural analogy and competitive inhibition (2).

In cancer, the stress has been upon the enzymes, but, after all, there are two factors, the enzyme and the substrate: the latter has been more or less neglected in cancer study. And it is the pabulum of cell growth. If the food supply of the cell (call it metabolites if you will) can be cut off, the cell will not grow any more than will plants deprived of their

nutrition. Many years ago one of us (McD) (3) drew the analogy of cancer to weeds in a field of grain where the ideal is to stop the growth of the weeds, so that they die, and not affect the corn. If the growth is stopped long enough the weeds will die. There is a term used in military parlance "interdiction" which means cutting the enemy's supply line, etc., behind him. Examples are the Alexander campaign in Italy which caused the surrender of the German Armies there and the present Air "Operation Strangle" in Korea cutting off the railroad and truck avenues to North Korea. So in cancer, if the supply can be "interdicted" the enemy will surrender.

In enzymatic parlance the supply is the substrate. So that the first search is for a chemical analogue of the substrate. Antagonists for a wide variety of metabolites have been investigated in recent years. The list includes many amino acids, vitamins, steroids, lipids, ions, and "growth factors" or preformed metabolites essential to the vitality of specific cells. In general, the list is notable for the absence of antagonists to glucose or sugars used as a universal source of energy by living cells. It is true that competition has been demonstrated between sugars which are normal substrates for the glycolytic process. The glycolysis of fructose by yeast is competitively inhibited by glucose (4) but only in the sense that the "inhibitor" is preferentially used by the enzymes in the glycolytic process, while glucolysis as a whole proceeds normally. A specific competitive inhibitor for glucose which was not itself metabolized, might offer a novel means of controlling vital processes.

It has been demonstrated that the typical *intact* cancer cell exhibits an unusual ability to utilize glucose by the process of anaerobic glycolysis with the elaboration of lactic acid (5). Furthermore, the admission of oxygen to such a glycolyzing system fails to interrupt the glycolytic process (negative Pasteur effect), a phenomenon foreign to almost all mammalian tissues. These facts strongly suggest that *anaerobic-type glycolysis* is a very important if not vital aspect of metabolism in neoplastic tissues. The obvious conclusion may be drawn that an analogue of glucose, inhibitory to anaerobic glycolysis, should prove to be preferentially inhibitory to the proliferation of cancer cells.

Another aspect of cancer metabolism, less clearly defined, in which a carbohydrate antagonist may be predicted to exert a profound influence, involves the nucleic acids. Both ribo- and desoxyribo-nucleic acids are essentially long-chain polymers of the N-glucosides of ribose and desoxyribose, bound together by phosphate ester linkages. The nucleic acids may be regarded as carbohydrate in nature. DNA is one of the characteristic substances of the very genes whose self-perpetuating aberrations must be responsible for neoplastic proliferation. A specific antagonist for desoxyribose or for DNA may be expected to produce spectacular, though as yet unpredictable changes in the behavior of rapidly-dividing cells.

A research program aimed at the control of cancer through the use of carbohydrate analogues as antagonists is now under way at this laboratory. The possibilities in this new attack on an old problem are enormous and many new facets will undoubtedly present themselves as the work progresses.

The initial attack has been through the use of 2-desoxy-D-glucose as an antagonist of blood sugar. The desoxy sugar is structurally identical with common glucose, except that the hydroxyl group on the second carbon atom has been replaced by hydrogen. Actual tests of the inhibitory action of 2-desoxy-D-glucose on the growth of tumors in experimental animals have produced some positive results, though no consistently effective treatment has been obtained to date.

Studies of the effects of 2-desoxy-D-glucose on the carbohydrate metabolism of simpler living systems comprise the subject matter of the papers which follow. These publications are now being placed before the scientific world with the hope that additional work in this field will be stimulated in other laboratories with a consequent hastening of the day when one of the greatest scourges of the human race will be brought under control.

REFERENCES

(1) E. WALDSCHMIDT-LEITZ AND E. MCDONALD, ET AL., "Ueber Enzyme in Tumoren. I.," *Z. physiol. Chem.*, **219**, 115 (1933).
(2) G. V. MARTIN, "Biological Antagonism," New York, Blakiston Company, 1951.
(3) E. MCDONALD, "The Chemistry of Cancer," *Lancet*, **2**, 1478 (1934); "Some Chemical Aspects of Cancer Research," *J. Chem. Ed.*, **9**, 820 (1932).
(4) R. H. HOPKINS, "Selective Fermentation," *Biochem. J.*, **25**, 245 (1931).
(5) F. DICKENS AND H. WEIL-MALHERBE, "The Metabolism of Normal and Tumor Tissue," *Cancer Research*, **3**, 73 (1943).

2-DESOXY-D-GLUCOSE AS AN ANTAGONIST OF GLUCOSE IN YEAST FERMENTATION

BY

FRANCIS B. CRAMER AND GLADYS E. WOODWARD

The investigation of the effects of 2-desoxy-D-glucose (2DG) on the fermentation of glucose by yeast, was prompted by the effects of the compound on animals. Occasional regression of Walker carcinoma 256 and more consistent termination of embryonic development in rats (experimental work as yet incomplete) had been observed to result from the intraperitoneal administration of near-lethal quantities of 2DG. Intact cancer cells and embryonic or placental cells possess in common a pronounced ability to metabolize glucose by anaerobic glycolysis. The glycolysis continues in the presence of oxygen whereas most tissues strongly favor a different metabolic pattern when oxygen is made available. Anaerobic glycolysis is apparently a significant and perhaps vital aspect of metabolism in the intact cancer cell. The importance of using the intact cell as well as tissue homogenates in glycolytic studies will become apparent from consideration of the results to be reported here.

Yeast was chosen for a study of the effect of 2DG on anaerobic glycolysis, since it is capable of carrying out glycolysis by a process which, in its initial stages, is almost identical with that used by mammalian tissues.

The structure of 2DG is identical to that of ordinary D-glucose with the exception that the hydroxyl group on the second carbon atom has been replaced by hydrogen. Almost all living cells possess enzyme systems capable of using glucose as substrate for chemical transformations which provide energy for vital processes. The structure of 2DG maintains the identity of those atomic configurations in glucose believed to be involved in attachment to some enzymes (1) and it is therefore to be anticipated that competitive inhibition of glucose-using enzymes by 2DG may occur.

The anaerobic fermentation of glucose by living yeast is shown to be strongly inhibited by 2DG. Evidence is adduced which indicates that 2DG, acting as a structural analogue of glucose, competitively inhibits an enzyme in the cell wall which under anaerobic conditions is essential to the passage of glucose into the interior of the cell.

MATERIALS AND METHODS

2DG was prepared as previously described (2).

Yeast cell suspensions in water were made directly from Fleischmann's pressed baker's yeast (starch-free) or from Fleischmann's "active dry yeast" obtained commercially. *Yeast extract* was a cell-free supernatant solution obtained by centrifugation from a 20 per cent aqueous suspension of the pressed yeast which had been treated in a Raytheon sonic oscillator (Type R–22–3) at 9000 cycles for 2 hours at 12–14° C.

Yeast hexokinase was prepared as a crude extract from toluene-autolyzed pressed yeast by a procedure essentially that of Kunitz and McDonald (3).

Fermentations were carried out in a Warburg manometric apparatus in an atmosphere of 5 per cent CO_2 in N_2. With living yeast, the fermentation mixtures contained, unless otherwise stated, only yeast cells, glucose or glucose plus 2DG, and water to a total volume of 2 ml. One of these components, depending upon the factor being studied, was placed in the side-arm of the flask. After equilibration at 37° C., the contents of the flask were mixed by tipping, and the CO_2 evolved was noted at intervals. The rate of fermentation was taken as the initial linear rate which followed the preliminary induction period. With yeast extract the experiments were carried out similarly except that the temperature was 30° C. When glucose was the substrate, adenosine-triphosphate (ATP), diphosphopyridine nucleotide (DPN), inorganic phosphate and magnesium ions were added as activators to the contents of the main part of the flask.

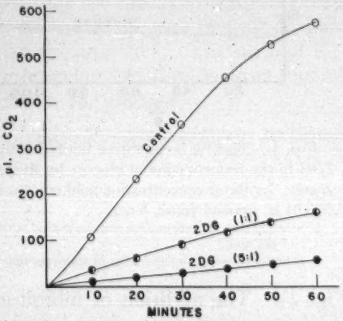


FIG. 1. Inhibitory effect of 2DG on the rate of fermentation of glucose by living yeast. Pressed yeast, 8 mg.; glucose, 0.01 M; 2DG, where added, 0.01 M and 0.05 M.

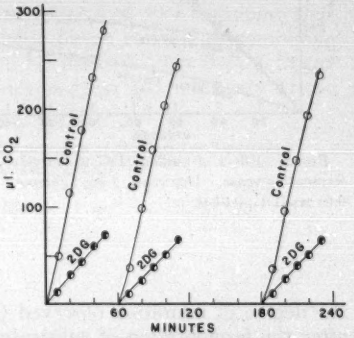


FIG. 2. Independence of degree of inhibition of fermentation rate on time of contact of 2DG with yeast before addition of glucose. Pressed yeast, 4 mg.; glucose, 0.05 M; 2DG, 0.05 M. Glucose added at 0, 60, and 180 min. after shaking of yeast control or of yeast plus 2DG in CO_2—N_2 at 37° C.

The hexokinase reaction was carried out in the Warburg apparatus by the method of Colowick and Kalckar (4), in which the liberation of one acid equivalent by the transfer of one phosphate from ATP to glucose results in liberation of one mole of CO_2 from bicarbonate in the medium.

RESULTS AND DISCUSSION

Experiments Using Living Yeast.—The incorporation of 2DG in yeast-glucose mixtures had a very marked inhibitory effect on fermen-

tation (Fig. 1). When substrate and inhibitor were present in a 1:1 ratio at the beginning of fermentation the rate of fermentation of glucose was reduced 78 per cent. The exposure of yeast to the inhibitor for various lengths of time before addition of substrate did not affect

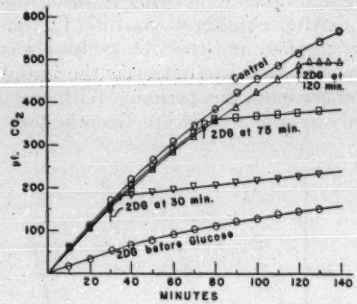


FIG. 3. Effect of adding 2DG to actively fermenting yeast. Dry yeast, 2 mg.; glucose, 0.01 M; 2DG, 0.01 M.

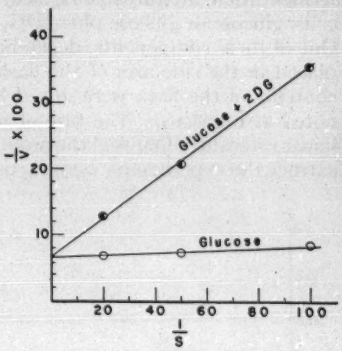


FIG. 4. Test for competitive inhibition by 2DG in the fermentation of glucose by living yeast. Inhibitor concentration held constant at 0.01 M; pressed yeast, 8 mg.

V = initial rate of fermentation expressed as μl. CO_2 per min.

S = initial substrate concentration in moles per liter.

the degree of inhibition observed (Fig. 2). The addition of inhibitor after the fermentation of substrate had proceeded for various lengths of time resulted in a steady inhibited rate within less than 5 minutes (Fig. 3). The greater inhibitory effect produced by the later additions of 2DG was due to the increased inhibitor-to-substrate ratio resulting from consumption of substrate by previous fermentation. Dependence of the degree of inhibition on the molar ratio of inhibitor to substrate was shown by the data in Table I. The degree of inhibition produced by 2DG, however, was not dependent on the absolute amounts of sub-

TABLE I.—*Dependence of Degree of Inhibition of Fermentation on Molar Ratio of 2DG and Glucose.*

Glucose Conc., M	Yeast Weight, mg.*	Glucose : Yeast (Weight Ratio)	pH (Initial)	Inhibition at 2DG : Glucose Ratio 1:1, %	Inhibition at 2DG : Glucose Ratio 5:1, %
0.10	40	0.90:1	6.3**	80	93
0.05	8	2.25:1	6.3**	77	92
0.05	4	3.60:1	6.3**	78	
0.01	8	0.45:1	6.3**	77	93
0.05	8	0.45:1	4.5***	80	
0.05	8	0.45:1	7.5***	80	

* Pressed yeast.
** Unbuffered.
*** 0.02 M phosphate buffers included in fermentation mixtures.

strate or of yeast, nor on the pH of the medium within the range 4.5 to 7.5. No fermentation occurred when 2DG alone was used as substrate.

The type of inhibition described was suggestive of the phenomenon of competitive inhibition of an enzyme-catalyzed reaction. A test for competitive-type inhibition described by Lineweaver and Burk (5) was used. Measurements of fermentation rates with and without added inhibitor were made with a series of substrate concentrations. The reciprocals of the initial fermentation rates ($1/V$) were plotted against the reciprocals of the substrate concentrations ($1/S$) (Fig. 4). Straight lines resulted, having a common intercept on the $1/V$ axis, but with an increased slope for the line depicting the inhibited reaction. The dissociation constant of the enzyme-glucose complex, K_S, was calculated, by the use of the Lineweaver and Burk equation (5), to be 2.7×10^{-3} and that for the 2DG-enzyme complex, K_I, 6.6×10^{-4} (assuming that all of the 2DG was present in anomeric forms capable of combining with the enzyme). The ratio of K_I to K_S indicates that the affinity of the enzyme for 2DG is over four times as great as the affinity of the enzyme for glucose.

TABLE II.—*Non-Inhibition by 2DG of the Fermentation of Fructose-1,6-Diphosphate (HDP) by Cell-Free Yeast Extract.*

Time, min.	Fermentation of 0.02 M HDP, µl. CO_2	Fermentation of 0.02 M HDP + 0.02 M 2DG, µl. CO_2
5	22	19
10	41	44
15	65	70
20	94	99
25	123	129
30	153	159

These findings confirmed the assumption that the inhibition of fermentation by 2DG was the result of a competition between the substrate and its structural analogue for attachment to an enzyme surface. It was apparent that the enzyme reaction affected was a rate-determining one for the entire fermentation scheme.

Experiments Using Yeast Extract.—The identity of the particular enzyme involved remained to be determined. For this phase of the work, cell-free yeast extracts were used. The cell-free extract obtained from sonicized pressed yeast was capable of fermenting fructose-1,6-diphosphate without the addition of activators. The fermentation rate was not affected by the addition of 2DG (Table II). Hence the enzyme inhibited in glucose fermentation by living yeast had to be one of those involved in the conversion of glucose to fructose-1,6-diphosphate.

Yeast hexokinase, generally considered to be the first enzyme involved in hexose fermentation, is known to act on three sugars, glucose, fructose, and mannose (3) and the amino-sugar, glucosamine (6). A new substrate for hexokinase, 2DG, may now be added to the list. In the presence of the enzyme, ATP phosphorylated not only glucose but

also 2DG alone or in admixture with glucose (Fig. 5). This indicated that 2DG should competitively inhibit the phosphorylation of glucose by hexokinase in the same way that glucose, fructose and mannose were shown to be mutually inhibitory to the hexokinase reaction by Slein, Cori and Cori (7). Since methods of analysis[1] for the sugar mixture involved were not yet available, this hypothesis was not tested directly.

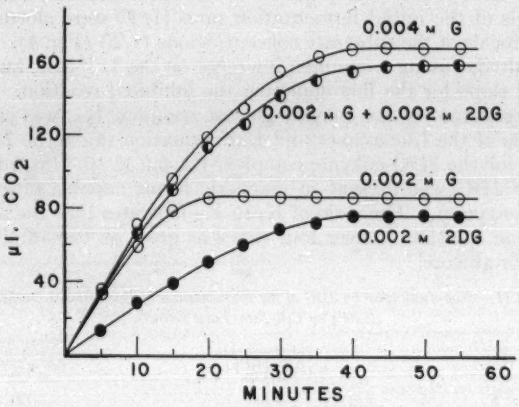


FIG. 5. Reactions of 2DG and of glucose (G) in the hexokinase-ATP system. ATP, 0.0087 M. [Total CO_2 equivalent to 0.002 M glucose (or 2DG) = 89.6 µl.]

The cell-free yeast extract was not capable of fermenting glucose at an appreciable rate unless activated by the addition of magnesium, inorganic phosphate, ATP, and DPN. The effect of added 2DG on the rate of fermentation of glucose by the fortified yeast extract is shown in Fig. 6. There was no inhibitory effect on the initial fermentation rate by 2DG in 1:1 or 2:1 molar ratio to glucose. This was in marked contrast to the effect of 2DG on fermentation of glucose by living yeast, where a 1:1 ratio of inhibitor to substrate produced an *immediate* 78 per cent reduction in rate of fermentation.

The rate of fermentation of glucose by fortified yeast extract was observed to gradually increase with time, whereas, in the presence of 2DG, the rate remained constant or fell off gradually. The explanation for this is to be found in the effect of ATP concentration on the rate of glucose fermentation by fortified yeast extract (Fig. 7). The curve for 0.002 M ATP represents a duplicate of the control experiment in Fig. 6. The use of more ATP resulted in a higher fermentation rate, and of less

[1] Mixtures of glucose and 2DG have been analyzed by determination of total reducing power before and after removal of glucose by yeast fermentation (8). Since the fermentation of glucose is inhibited by 2DG with increasing efficiency as the molar ratio increases, this method is open to considerable question.

ATP resulted in a lower rate. It has been shown above that ATP was consumed by 2DG in the presence of hexokinase. Hence, the addition of 2DG to a fermentation mixture of fortified yeast *extract* and glucose will result in a lowering of the ATP concentration with a consequent reduction in fermentation rate over a period of time.

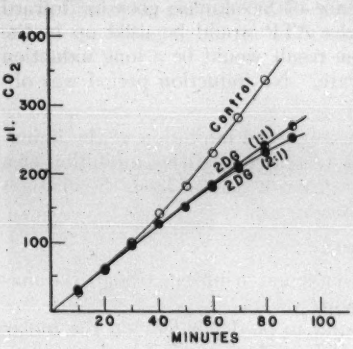


FIG. 6. Effect of 2DG on the rate of fermentation of glucose by fortified cell-free extract from sonicized yeast. Yeast extract, 0.5 ml.; ATP, 0.002 M; DPN, 0.001 M; Na-K phosphate buffer (pH 6.5), 0.01 M; $MgCl_2$, 0.01 M; glucose, 0.02 M; 2DG, where added, 0.02 M and 0.04 M.

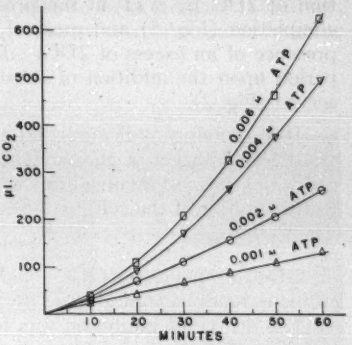


FIG. 7. Effect of ATP concentration on the rate of fermentation of glucose by fortified cell-free extract from sonicized yeast. Glucose, 0.02 M; yeast extract, 0.5 ml.; DPN, 0.001 M; Na-K phosphate buffer (pH 6.5), 0.01 M; $MgCl_2$, 0.01 M.

CONCLUSIONS

There is a fundamental difference between the effect of 2DG on the rates of fermentation of glucose by living yeast and by cell-free yeast extract. In the case of living yeast, 2DG acts strongly to competitively inhibit an enzyme which catalyzes a rate-determining step in the fermentation scheme. In the case of (fortified) yeast extract, 2DG does not show inhibition initially and the very slight inhibitory effect observed later may be explained by the consumption of ATP in a "one-way" reaction.

The difference between the two systems can be explained in terms of the presence or absence of the intact cell wall. The specific rate-determining enzyme involved must be located in this cell wall. The enzyme must be essential to the passage of glucose into the cell under anaerobic conditions, since the fermentation of glucose by the enzymes within the cell (i.e., in yeast extract) is not inhibited by 2DG.

Consumption of ATP by 2DG within the yeast cell is an improbable mechanism of the inhibition of living-yeast fermentation for the following reasons:

1. Inhibition due to ATP depletion would not be expected to follow the laws of competitive inhibition.

2. The inhibitory effect produced by adding 2DG to an actively fermenting system is exerted immediately (Fig. 3) and remains constant over a period of time (Fig. 1).

3. Exposure of yeast to 2DG before adding the substrate does not increase the amount of inhibition produced (Fig. 2). The phosphorylation of 2DG by ATP in the presence of hexokinase goes far toward completion (Fig. 5) and most of the ATP would be used up in the presence of an excess of 2DG. The result would be a long induction period upon the addition of substrate. No induction period was observed (Fig. 2).

It is therefore indicated that the observed inhibition of the living-yeast fermentation of glucose is due to the competitive inhibition of a cell-wall enzyme,[2] involved in the transfer of glucose from the medium to the interior of the cell, by 2-desoxyglucose.

SUMMARY

2-Desoxy-D-glucose (2DG) has been found to inhibit strongly the anaerobic fermentation of glucose by living yeast.

The degree of inhibition was found to be dependent on the molar ratio of substrate to inhibitor, but independent of the absolute quantities of substrate or yeast and of pH of the medium between 4.5 and 7.5. The effect followed the laws of competitive inhibition of an enzyme.

The initial rate of fermentation of glucose by fortified cell-free yeast extract was not inhibited by 2DG. The inhibitory effect of 2DG on yeast fermentation was lost with the removal of the yeast cell wall.

The evidence indicates that 2DG, acting as a structural analogue of glucose, competitively inhibits the action of a cell-wall enzyme which is essential for the transport of glucose into the cell, thus inhibiting the overall rate of fermentation by living yeast.

A new substrate for yeast hexokinase has been discovered through the observation that the adenosine triphosphate-hexokinase system phosphorylated 2-desoxy-D-glucose.

REFERENCES

(1) A. GOTTSCHALK, *Adv. in Carbohydrate Chem.*, **5**, 49 (1950).
(2) F. B. CRAMER, J. FRANKLIN INST., **253**, 277 (1952).
(3) M. KUNITZ AND M. R. MCDONALD, *J. Gen. Physiol.*, **29**, 393 (1945–46).
(4) S. P. COLOWICK AND H. M. KALCKAR, *J. Biol. Chem.*, **148**, 117 (1943).
(5) H. LINEWEAVER AND D. BURK, *J. Am. Chem. Soc.*, **56**, 658 (1934).
(6) P. T. GRANT AND C. LONG, *Biochem. J.*, **50**, xx (1952).
(7) M. W. SLEIN, G. T. CORI, AND C. F. CORI, *J. Biol. Chem.*, **186**, 763 (1950).
(8) W. FREUDENBERG, *J. Biol. Chem.*, **99**, 647 (1932–33).
(9) A. ROTHSTEIN, R. MEIER, AND L. HURWITZ, *J. Cellular Comp. Physiol.*, **37**, 57 (1951).

[2] The presence of an enzyme system, in the yeast cell wall, which carries out a chemical alteration of glucose and is essential to the entrance of glucose into the cell under anaerobic conditions has recently been postulated by Rothstein (9) as a result of his observations on the inhibition of glucose fermentation by uranyl ions. The evidence presented in the present paper is in complete accord with this concept.

THE INFLUENCE OF 2-DESOXY-D-GLUCOSE ON THE GROWTH OF EMBRYONIC CHICKEN-HEART FIBROBLASTS IN TISSUE CULTURE

BY

J. O. ELY, F. A. TULL AND J. A. HARD

2-Desoxy-D-glucose (2DG) was found to be a strongly inhibitory, but not a killing agent for embryonic chicken-heart fibroblasts in tissue culture.

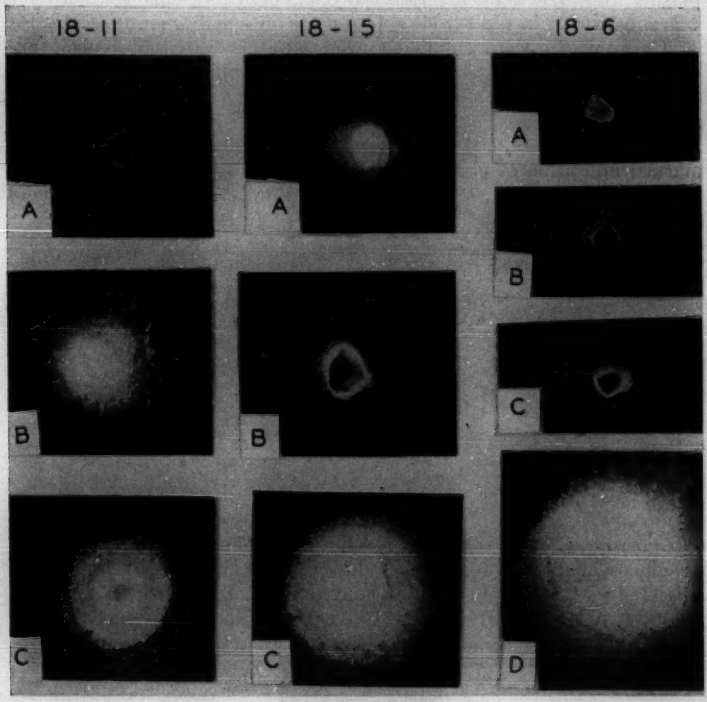


FIG. 1. Effect of 0.04 per cent 2DG on tissue cultures of embryonic chicken-heart fibroblasts. Magnification 11×.

18-11 *a.* 0–41 hours 2DG.
b. 41–89 hours glucose (substituted for 2DG).
c. 89–137 hours glucose.

18-6 *a.* 0–41 hours 2DG.
b. 41–89 hours 2DG.
c. 89–137 hours 2DG.
d. 137–377 hours glucose (substituted for 2DG).

18-15 *a.* 0–41 hours glucose.
b. 41–89 hours glucose.
c. 89–137 hours glucose.

METHODS

Cultures of 8–10 day old chicken embryo heart fibroblasts were grown either on glass coverslips or in carrel flasks. The culture medium when carrel flasks were used consisted of chicken plasma 3 parts, tyrode solution 4 parts, 8–10 day old chicken embryo extract 2 parts, 0.05 per cent phenol red solution 1 part; the sugar concentration was 0.04 per cent. When coverslip cultures were used the medium consisted of 1 part tyrode solution, 1 part embryo extract and 2 parts chicken plasma; the sugar concentration was 0.025 per cent. 2DG, when used, was substituted in the tyrode solution for an equal amount of glucose.

Carrel flasks were employed when it was desired to replace one medium containing sugar by another medium containing the other sugar.

RESULTS

Incorporation of 0.04 per cent of 2DG in the culture medium caused marked inhibition of growth as indicated by the surface area covered by the cultures (Fig. 1, 18–11, a and 18–6, a, b, c) and a concentration of

TABLE I.—*Influence of Desoxyglucose on Mitosis of Chicken Embryo Fibroblasts in Tissue Cultures.*

Sugar	Concentration of Sugar, %	Age of Culture, hours	Mitotic Index, %
1. Glucose	0.04	45	2.7
2. Desoxyglucose	0.04	45	0.3
3. Glucose	0.04	45	0.4
Desoxyglucose	0.04		
4. None	—	45	2.0
5. Glucose	0.04	22	2.6
6. Desoxyglucose	0.04	22	0.0 (1 culture)
7. Glucose	0.04	22	0.3
Desoxyglucose	0.04		
8. None	—	22	2.5

0.025 per cent 2DG greatly reduced the mitotic index (Table I). When the glucose medium was substituted for the 2DG medium after 41 hours (Fig. 1, 18–11, b, c), or after 137 hours (Fig. 1, 18–6) growth occurred. Incorporation of 0.04 per cent 2DG with 0.04 per cent glucose, or of

FIG. 2. Effect of 0.04 per cent 2DG on tissue cultures of embryonic chicken-heart fibroblasts for 0–68 hours. Magnification 11×.

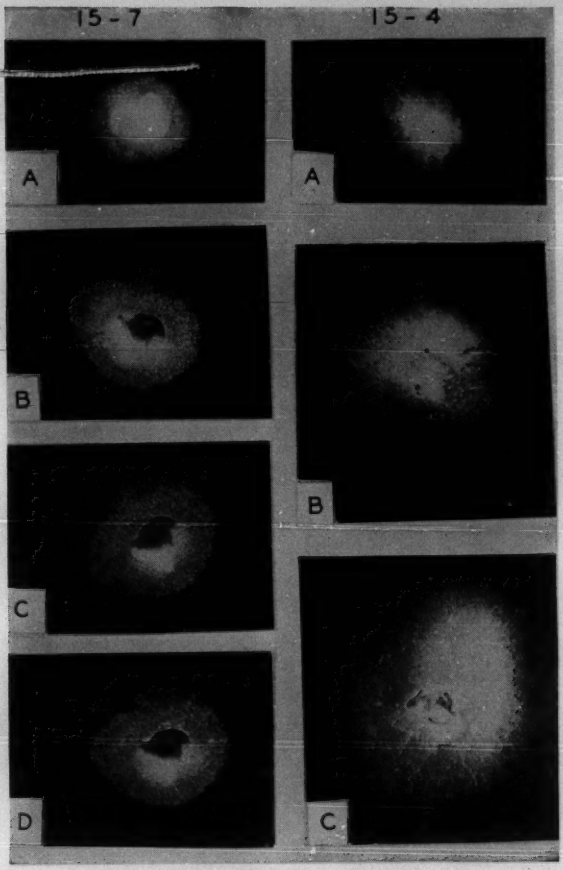


FIG. 3. Effect of 0.04 per cent 2DG on tissue cultures of embryonic chicken-heart fibroblasts. Magnification 11×.

15–7 *a.* 0–64 hours glucose.
b. 64–135 hours 2DG (substituted for glucose).
c. 135–183 hours 2DG.
d. 183–280 hours 2DG.

15–4 *a.* 0–64 hours glucose.
b. 64–135 hours glucose.
c. 135–183 hours glucose.

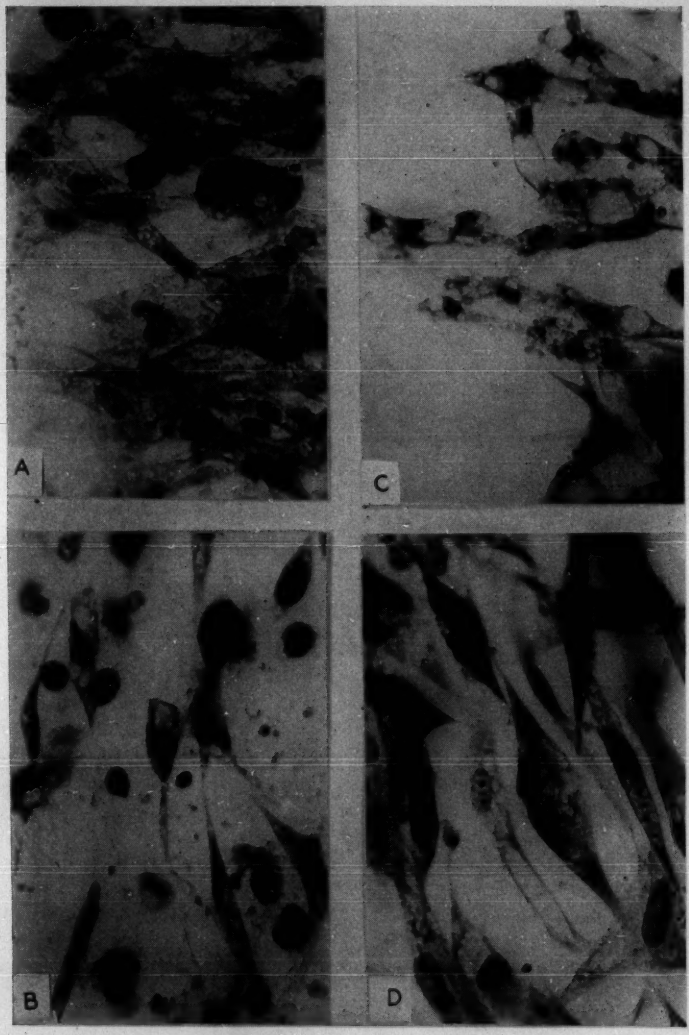


FIG. 4. 41-hour cultures fixed in Bouin's Fluid stained with iron hematoxylin. Magnification 525×.

a. Glucose	0.025 per cent.
b. 2DG	0.025 per cent.
c. Glucose and 2DG	0.0125 per cent of each.
d. No sugar.	

0.02 per cent 2DG with 0.02 per cent glucose in the medium inhibited growth to approximately the same extent as did 2DG alone.

Migration of the cells of cultures in the presence of 2DG was limited and the cultures appeared coarse (Fig. 2). When first grown for a preliminary period in the presence of glucose, and then in a medium containing 2DG, cultures enlarged very little and appeared to become thickened, coarse in appearance and sharp in outline (Fig. 3, 15–7), compared to cultures with glucose in the medium (Fig. 3, 15–4).

Cytologically the cells of cultures grown in the presence of 2DG with or without glucose present, became highly vacuolated (Fig. 4, b, c) presumably due to fat accumulation, in comparison to those of cultures of the same age grown with glucose (Fig. 4, a) or with no sugar in the medium (Fig. 4, d).

The effect of 2DG, obviously, was inhibition of growth rather than killing since slight growth occurred and a low mitotic index was found in cultures grown in a medium containing this sugar, and cultures later grew at an apparently normal rate after glucose was substituted in the medium for 2DG.

New Fixer for Photographic Film.—Elimination of the acrid fumes usually associated with acid hypo solutions has been achieved by Du Pont in its new 18–F Universal Fixer now being introduced. This is a single-powder hardening fixer for all types of photographic films and papers, and has the added advantages of mixing ease and long service life.

The chemical that does the trick is an aromatic, an oil of lavender derivative, that masks out the strong acetic acid and sulphite odors, thereby making the photographic darkroom a more pleasant place to work. This odor deemphasis is particularly desirable where processing is done in office buildings, in homes, or where darkrooms adjoin studios or reception rooms.

Trans-Atlantic Television.—The man who helped make possible the first trans-Atlantic radio broadcast 45 years ago predicted recently that a form of trans-oceanic television may not be far off.

Rounding out a half-century of inventing with the General Electric Company's General Engineering Laboratory, Dr. Ernst F. W. Alexanderson suggested a method whereby events in Europe or other foreign countries might be telecast in this country within an hour or two after they take place.

He explained that such events could be recorded on motion picture film and the film then transmitted by radio facsimile to America.

"When facsimile movies are received here, the film could be quickly processed and then televised from broadcasting stations in the same manner as now done on television newscasts," he said.

The suggested system was first used by Dr. Alexanderson during early television demonstrations in 1926, in which he slowed down transmission of pictures by facsimile and then by processing the films, speeded them up to motion picture projector rate for television broadcasts.

One of the nation's most prolific living inventors, Dr. Alexanderson has a total of 320 patents.

Listed among his numerous patents are the famous "alternator," which gave America its start in the field of radio communication. This high-frequency machine for powerful continuous wave transmission resulted in the first trans-Atlantic radio broadcast on Christmas Eve, 1906. He is also responsible for development of the first home television receiver, which he demonstrated for the first time in 1926.

The Swedish-born inventor has many other patents in radio telephony, radio telegraphy, motors, generators, electric gunnery control, electric ship propulsion, electric traction, power transmission, electronic circuits, vacuum tubes for radio and power rectifiers, and still others in television.

Since his first invention in 1904, Dr. Alexanderson has averaged one patent every seven weeks during 45 years of active service with General Electric. Since his retirement from the company in 1948, he has received 11 more patents, while engaged by the company in a consulting capacity.

Plastic piping will be installed in new minesweepers now being built for the Navy, saving about two tons of copper per ship. Cost of plastic pipe in 2-in. size is about 70 cents per foot, compared with $1.55 for copper-nickel and $2.25 for stainless. Bureau of Ships announced that in an 8-month test aboard a destroyer escort, plastic pipe held up where metallic piping gave way due to corrosion, shock, and vibration. Pipe is fiber glass cloth bonded with synthetic resin, can be patched with fiber glass tape impregnated with plastic.

BOOK REVIEWS

EQUIVALENT CIRCUITS OF ELECTRIC MACHINERY, by Gabriel Kron. 278 pages, drawings, 15×24 cm. New York, John Wiley & Sons, Inc., 1951. Price, $10.00.

Engineers have developed many types of models to help themselves in visualizing and analyzing the performance of electrical machinery. The significance of these models arises out of the economic importance associated with high performance in rotating machinery. Likewise, many of the technical problems encountered in the improvement of rotating machinery performance are such that one would hesitate in attempting a direct analytical solution.

The type of model employed has varied between the strictly mathematical (equations), the vector diagram, the locus plot and the equivalent circuit. The circumstance and complexity of the problem have usually dictated which of these was to be used. Not infrequently the past practice of the individual engineer is the sole reason for a selected procedure.

In his latest book Kron sets forth the results of a decade in which he has studied mathematically developed equivalent circuits to uncover their fundamental similarities. This work has resulted in a non-mathematical representation of models. Although the term "tensor" is not used in this representation, and although there are no matrices of mathematical symbols, the tensor idea is expressed by means of electrical circuit symbols. Multimesh circuits of electrical impedance combinations are used as models of machines and interconnections of machines.

The author shows how currents, fields, and torques can be computed directly by the use of their equivalent models. In preparation for this step the principles are given for determining a representative model and for insuring that true equivalence has been achieved by it.

The reviewer feels that there is much elegance in Kron's methods and that the research directed toward the development of electrical machinery may be furthered by their application. However, this book could be written in a more understandable form. Had the volume included more words of explanation in all sections, it would have conveyed more information and would have brought the author's ideas within the reach of a greater number of engineers. As it stands, a great deal of patience is required on the part of the reader. One is forced to read and reread the author's brief statements in order to draw out the numerous implications. Considering the prodigious amount of work done by the author in arriving at his generalizations, it is clear that much of the intermediate and practical work has been omitted. The inclusion of this material for illustrative purposes would have made a better presentation. The book's short length, its summary form and lack of any kind of index places it in the catagory of a monograph rather than a text or reference book.

C. W. HARGENS

ASTROPHYSICS, edited by J. A. Hynek. 703 pages, illustrations and diagrams, 16×24 cm. New York, McGraw-Hill Book Co., Inc., 1951. Price, $12.00.

In the preface of this volume Dr. J. A. Hynek, its editor, says, "On the occasion of the fiftieth anniversary of the Yerkes Observatory casual conversation centering around the appropriateness of a review of progress in astrophysics during the half century since the founding of the observatory—progress in which the Yerkes Observatory played an integral role—led to the preparation of this volume." Therein lies the reason for this book.

While the symposium was conceived as an exploration of a problem in astrophysics addressed to a "hypothetical first year graduate student . . . by no means a specialist" it has resolved itself into a series of technical papers which, for a brief instant, touch upon the historical background and then move forward into an up-to-the-minute discussion of the subject with the impact of an express train.

The volume consists of fourteen articles arranged in four general groups:

1. *Spectroscopic Astrophysics.* Contributing to this section are found the leaders in this field—Struve, Morgan, Swings, Keenan, Aller, and Stromgren. It deals with the analysis, interpretation, and classification of Stellar spectra.

2. *Physics of the Solar System.* This section is the most exciting for this reviewer. Again top authorities in the respective fields are the contributors. Edison Pettit writes on the sun, Dr. Bobrovnikoff writes on comets and Kuiper covers the origin of the solar system. Significant work had been done on this last subject and it will be high on the list of projected problems.

3. *Physics of Binary and Variable Stars.* Contributors to this section are Drs. Van Biesbrock, Hynek, Payne, Gaposchkin, and the late Newton L. Pierce. On the whole the subject matter is comprehensive but this reviewer feels the subject of Eclipsing Binaries has been slighted. Eclipsing Binaries has become a major branch of Astrophysics with many ramifications. These should have been included in this section.

4. *Physics of Cosmic Matter.* Two giants in their respective fields have contributed to this section. Dr. Jesse L. Greenstein is responsible for a magnificent treatment of interstellar matter and Dr. S. Chandrasekhar has written about "The Structure, the Composition, and the Source of Energy of the Stars."

"Astrophysics" is a major contribution to astronomy and will prove of inestimable value to the serious student of the subject. Because of inexorable advances in the above topics this volume is not and cannot be the final word. It is conceivable that it may be the beginning of a 10-year series to be published every decade to furnish the scholar with a comprehensive and current review. If this should be the case then the topics may be expanded and other specialists—not associated with the Yerkes Observatory—included as contributors.

The price of "Astrophysics," $12.00, is very high and may be effective in keeping it out of the reach of the very ones who need it most—the serious graduate students.

I. M. LEVITT

TELEVISION ENGINEERING, by Donald G. Fink. Second edition, 721 pages, illustrations, 16×24 cm. New York, McGraw-Hill Book Co., Inc., 1952. Price, $8.50.

This is the second edition of the author's book entitled "Principles of Television Engineering," published in 1940. Since the material presented in the initial edition has been obsolete for some time, the author has superseded it with his new work.

A concise estimate of the amount of material contained in the second edition may be obtained with the aid of some statistics. For example, approximately 94 per cent of the original text has been rewritten and augmented by 200 pages of new information. A little more than 20 per cent of the 312 original illustrations were retained and 440 new ones were added. As a result of this extensive revision, the second edition is practically all new material.

As befitting an attempt to present subject matter relating to a field so expansive as television, the author has delved into the philosophical, as well as the practical, aspects of television circuits, components, and systems. The introductory chapters present the fundamental aspects of television, and the remainder are devoted to the meat of the matter. Not, however, without including some "gravy" to make it easy reading.

The reader may find such thought-provoking statements as the author's conjecture that ". . . television engineering is necessarily a science of compromises." If the knowledge gained in this manner is capable of being applied systematically then, by definition, it is a science. By the same reasoning, if this type of engineering is comprised of skill and the efficient use of knowledge, then it may well be more an art than a science.

This serves to typify the author's presentation of the subject matter. For one thing, it seems to blend with the increasing interest in the philosophical aspects of a very important and popular field of endeavor. While the author has not attempted to be rigorous in his presentation of mathematical relationships, he should be able to hold the interest of those more technically inclined. Discussion exercises and problems (with answers) that can be solved with the aid of the fundamental relationships given in the book have been included. More-

over, graphs, charts, cross sectional views, and block diagrams accompanied by their schematics (with values of circuit components) are liberally interspersed throughout the text.

It is unlikely that this book was intended to serve as a text, although it could, for some courses of study. However, it can best serve the purpose of a reference book, especially in view of the fact that a fairly comprehensive bibliography listing books and journals, by topic, follows each chapter. Even though the author has assumed a knowledge of the elementary principles of vacuum tube circuits, this book should appeal to the well-versed layman as well as the technical man.

E. A. FASKO

STATISTICAL DESIGN AND ANALYSIS OF EXPERIMENTS, by D. S. Villars. 455 pages, illustrations and charts, 15×22 cm. Dubuque, Wm. C. Brown Co., 1951. Price, $6.50.

In writing this book—the fourth major one to come out in recent years on the subject of design of experiments—Mr. Villars follows the lead of his predecessors, Fisher, Cochran and Cox, and Mann, by emphasing the tremendous power of factorial designs and variance analyses.

Actually, he goes much farther than this. He gives a thorough comprehensive presentation of the whole field of small sample statistics from the "t" test and basic distributions through components of variance, analysis of variance, covariance, latin squares, control charts, and sequential analysis. Not only does he present the formulae and their uses, but also the assumptions and implications of their uses. The value of the book is greatly enhanced by his non-mathematical presentation throughout, with mathematical proofs in the back.

Since the author learned most of his statistics from people of the British school, he has a lot to offer the experimentalists who were trained in this country, where the use of small samples has come into prominence only in the past ten years or so.

Two new concepts are developed in this book. One is the subject of replication degeneracy or incomplete replication of one or more of the variables under test. The other is a rational method for determining the correct method of analyzing variance.

One minor criticism of this work that might be made is the organization of the material. Throughout the early part of the book, various terms are used and not explained fully until later sections. This causes some slight irritation despite the skillful use of cross references and the glossary of terms in the appendix.

Although the preface states that this book was written as an introductory book to the field of statistical methods, its greatest use probably will be as a reference among the people already in the field.

EDWARD FRITZ

TENSOR ANALYSIS, by I. S. Sokolnikoff. 335 pages, drawings, 15×23 cm. New York, John Wiley & Sons., Inc., 1951. Price, $6.00.

If an author can show that a mathematical tool achieves simplicity, or if he can demonstrate its effectiveness, his presentation will be hailed by engineers, physicists, and mathematicians alike. We like to see the applicability of mathematical methods because we are naturally looking for better and more powerful analytical tools. This review is intended to consider in particular the utilitarian aspects of Sokolnikoff's new book on tensor analysis, for this is not a new subject and the volume will be used primarily as a textbook.

Approximately the first third of the volume is devoted to reviewing and developing tensor theory, while the remainder treats the application of tensors to problems in geometry and mechanics. This book strikes a middle course between the specific applicability of Kron's "Tensor Analysis of Networks" and the recently published "Tensor Analysis for Physicists" by Schouten (Amsterdam).

Several problems are given following each development of a particular thought, so that the student can test his own grasp of the analytical method described. Although these problems generally serve as an exercise relating to the teaching point, they would not exactly be called practical illustrations. This criticism applies less strongly to the sections on mechanics than to those dealing with geometry.

As is characteristic of Sokolnikoff's previous writings, the style and clarity of this text make it comparatively comfortable to follow considering its subject matter. Occasional brief references are made in an interesting way to supplementary subjects such as the history and philosophy of the "least action" principle. It is also the opinion of this reviewer that the qualities of ease and simplicity in writing usually characterize authors who have an orderly understanding of their subject, and this feature has been at least partly responsible for Sokolnikoff's success.

The pace of this text is rapid and the reader is expected to have a broad mathematical background so that the numerous mathematical principles to which the author alludes by name of the inventor will be recognized. From these observations, and also because the problems treated are of an academic type, the book is assessed as being of only occasional interest to engineers, of some appeal to physicists, but primarily for mathematicians.

C. W. HARGENS

FUNDAMENTALS OF ATOMIC PHYSICS, by Saul Dushman. 294 pages, illustrations and tables, 16×24 cm. New York, McGraw-Hill Book Co., Inc., 1951. Price, $5.50.

"Fundamentals of Atomic Physics" is the outgrowth of notes prepared by the author as an aid in a course in "new developments in the physical sciences" given for high school teachers of science at Union College in Schenectady and at Case Institute of Technology in Cleveland.

The author, a skillful expositor, employs a unique and most unorthodox approach in opening his book. The first two chapters are devoted to a history of physics and mathematical techniques and a discussion of atomic physics. Very few authors are considerate in laying a foundation for the reader to build on. In this book the reader is taken by the hand and guided through the preliminary steps necessary for the fuller understanding of the subject matter.

Included in this book are thorough treatments of the kinetic theory of gases, photoelectric effect, X-rays, the Bohr theory of the origin of spectral lines. From these classical treatments the author goes to the most recent advances in the investigation of nuclear phenomena, the development of atomic energy, and the evolution of particle accelerators.

The profusion of illustrations, footnotes, and quotations helps the reader through the more difficult parts of the text.

Careful reading discloses an almost complete lack of errors which is unusual in a first edition of this type. One error was found and that was in the diagram on page 124. The 8000 angstroms indicated on the top scale should be 6000 angstroms if the H alpha line is where it should be.

Though pointed for the engineer and the technical man, "Fundamentals of Atomic Physics" is unqualifiedly recommended for anyone who would like to acquire a working knowledge of this new science. It is a most welcome addition to the library of the scientist.

I. M. LEVITT

PRACTICAL ELECTRON MICROSCOPY, by V. E. Cosslett. 299 pages, illustrations, 16×26 cm. New York, Academic Press, Inc.; London, Butterworths Scientific Publications, 1951. Price, $5.50.

The science of electron microscopy has been growing rapidly for about twenty years. Numerous treatises have been written—some are mathematically too unwieldy from an operational viewpoint, some are instruction books emphasizing information on the instrument and excluding techniques of sample preparation and photography, and some (on sample preparation) are too restricted in scope. In this book, Professor Cosslett has succeeded in presenting a text which serves as an up-to-date guide for most persons in this field. It should be especially useful to those investigators who have not had specialized training in physics, but are interested in the applications of the electron microscope.

The author covers a wide range of subjects, most of which are pertinent to the optimum operation of the microscope. Because of this diversity, some items are given but a cursory

treatment; for a fuller discussion of subject matter, the reader is provided with an excellent bibliography of 290 important original papers. This bibliography, together with the author and subject indices, makes this book a useful reference guide, although it does not meet the need for a text which covers the subject as thoroughly as V. Borries' "Ubermikroskopie."

For information on sample preparation, Professor Cosslett recommends that his book be supplemented by The Royal Microscopical Society's "The Practice of Electron Microscopy." This publication sometimes stresses cumbersome methods, which Cosslett commendably circumvents through the use of physical plausibility arguments, thus enhancing the physical intuition of the microscopist.

"Practical Electron Microscopy," in the reviewer's opinion, should be on the shelf of every neophyte electron microscopist.

J. WEINMAN

BOOK NOTES

IDENTIFICATION OF TEXTILE MATERIALS. Third edition, 94 pages, illustrations and charts, 16×26 cm. Manchester, The Textile Institute, 1951. Price, 10/6d.

Considerably revised and extended, this third edition includes a short description of the chemical and physical properties of textile fibers—natural, animal and vegetable, manufactured —regenerated cellulose, cellulose acetate, synthetic polymer, and miscellaneous (asbestos, glass and paper yarns).

ELECTRICAL COMMUNICATIONS EXPERIMENTS, by Henry R. Reed, T. C. Gordon Wagner and George F. Corcoran. 458 pages, illustrations, 15×24 cm. New York, John Wiley & Sons, Inc., 1952. Price, $6.75.

The text is designed as a laboratory manual. Sixty fundamental experiments are divided into d-c. and a-c. fundamentals, and long lines, engineering electronics and radio engineering requiring the use of standard measuring equipment and components. The theoretical discussion precedes each experiment and consideration is given to the probable maximum involved in the physical measurements. An excellent guide to the student learning the techniques of electrical measurements.

ORGANIC SYNTHESIS: AN ANNUAL PUBLICATION OF SATISFACTORY METHODS FOR THE PREPARATION OF ORGANIC CHEMICALS. VOLUME 31, 1951. 122 pages, illustrations, 15×24 cm. New York, John Wiley & Sons, Inc., 1951. Price, $2.75.

Volume 31 of "Organic Synthesis" gives procedures for the preparation of compounds and equations for the reactions for 42 different organic compounds. Preparations are listed alphabetically under common name and "Chemical Abstracts" indexing name. All procedures are carefully checked. The subject index includes all materials listed in Volumes 30 and 31.

PRINCIPLES OF RADIO, by Keith Henney and Glen A. Richardson. Sixth edition, 655 pages, illustrations, 15×22 cm. New York, John Wiley & Sons, Inc., 1952. Price, $5.50.

This sixth edition of "Principles of Radio" is a completely revised and rewritten text. The fundamentals of basic radio circuits are carefully presented along with practical problems and many detailed numerical calculations. Chapters are devoted to frequency modulation, television, radar, wave shaping, and electronic test equipment. It is a practical text aimed to be used by persons learning the principles of radio without the aid of a teacher.

YEAR BOOK OF THE TEXTILE INSTITUTE AND LIBRARY CATALOGUE NO. 4, 1951–1952. 338 pages, 15×24 cm. Manchester, The Textile Institute, 1951. Price, 15/.

The fourth Year Book contains more reference material than published in previous editions, including: the listing and sources for 150 films on textile subjects; a catalogue of authors of 2000 publications in the Institute Library; and a complete list of Patrons and members of the Institute. The Year Book also contains a section devoted to education in textile technology, standardization terms, and definitions.

CURRENT TOPICS

Uranium Mining in the Colorado Plateau.—In the heart of the Colorado Plateau, a kidney-shaped area covering roughly 50,000 square miles at the four adjoining corners of Colorado, Utah, Arizona, and New Mexico, the quest for uranium is on in earnest these days. Modern miners are finding a crumbly yellow substance called carnotite from which are derived traces of uranium. Specially equipped International trucks are transporting the ore down the western slopes of the Rockies to processing mills. Refining extracts uranium from the carnotite scraps and processes it into black powder; the powder is collected and shunted to mysterious destinations for further secret processing.

The quest is an intricate one. Uranium's a tricky thing to count on; it does not run in veins. It runs in pockets. The quest takes prospectors to the rims of canyons and tops of mesas. To get four tons of the desired metallic element you have to dig a mine a mile deep and process something like two and one-half million tons of rock.

There's also the matter of delivering ore mined on the plateau to one of seven widely scattered mills at Grand Junction, Rigle, Naturita, Uravan, and Durango in southwestern Colorado, and Monticello and Rite in southeastern Utah. Big six-wheel trucks with hypoid-gear axles are the modern packhorses on this job. For every sixteen miles they travel with ore loads, fifteen miles are over hardly negotiable trails. The roads—if there are roads—are rocky routes blasted out by dynamite, or they are simply vague tracks, wandering up and down a great mass of talus and debris.

Mere trails, narrow defiles that wind between the mountains, primitive rock and dirt roads, grades that range from 15 to 30 per cent, breath-taking altitudes from 7000 to 13,000 ft. above sea-level, scorching desert heat, winter temperatures sometimes reaching 25 degrees below zero—all are encountered in this rugged task. The surfaces and inclines are dusty washboards in summer and slippery nightmares in winter. There is no room for passing. Rainstorms and melting snows in spring wash out trails. In the summer, the region dries into a dustbowl and landslides are a constant peril all seasons.

In spite of its inhospitality to man, the Colorado Plateau produced uranium for the atomic bombs that brought an end to World War II. Now scientists, engineers, and statesmen look to Plateau for the invaluable isotopes for medical and chemical research, bombs for national defense, and future power for industry. Colorado-produced ores have placed the United States in second place among free nations in the production of uranium, according to the Atomic Energy Commission.

Most of the carnotite is transported on International LF-210 and LF-190 model trucks, equipped with 16-ton-capacity dump bodies. They are driven to the mines, loaded, and then driven back to mills over rocky, sometimes indistinct, way. The braking test is severe—there are repeated stops because of obstacles. The transmission takes real punishment; the driver must continually run the range of the truck's five speeds forward, shifting on every curve and new rise as the truck roars upgrade.

A measure of how the uranium industry has boomed is the increase in the number of mines being worked, all by private business. They have grown from sixteen to approximately 250 since April 1948. Where 100 men formerly worked on these diggings, there are now over a thousand. Instead of a handful of trucks, scores of independent truck owners—operating from three to ten carriers each—are hauling tons of ore mined in the area.

The United States is still in the stage of rounding up uranium deposits. Experts say chances are slim of finding a mine like the fabulous Shinkolobwe in the Belgian Congo which yields the so-called pitchblende, richest uranium ore. There are pitchblende deposits at the second most important Anglo-American-Canadian arsenal at Eldorado in Canada's Northwest territory, but the Soviet Union has a rich grab in high-grade uranium deposits in the Erzgebirge region of Czechoslovakia and German Saxony. Meanwhile, this nation's future in atomic energy lies in revolutionizing methods of prospecting and processing low-grade carnotite ore.

There is no magic glass slipper for this Cinderella of metals. But American business know-how—as personified by the mining engineers, prospectors, miners, and truck drivers—is well on the way to leading this nation into uranium self-sufficiency.

Pipe Line for Nickel Ore Concentrate.—A 7½-mile pipeline has been completed by The International Nickel Company of Canada, Limited through which the bulk concentrate from 3,650,000 tons of nickel-copper ore is being pumped annually from its newly-built Creighton concentrator to its reduction plants at Copper Cliff. It is believed that never before has concentrate been carried by pipeline in such quantity over such a distance.

In making the announcement R. L. Beattie, Vice-President and General Manager, said: "The pipeline reduces the time required to transform nickel ore into refined nickel. Further, the economies resulting from pipeline transportation, together with other economies, have made possible the working of lower-grade Creighton ore which is now contributing to the availability of nickel for the defense production needs of the free world."

Part of International Nickel's $130,000,000 program of underground mine expansion, the pipeline artery has been integrated into an elaborate pipeline system which also carries tailing, or waste material, from both Creighton and Copper Cliff to a disposal area midway between the two plants.

The system has 12 miles of trestle, at some points 65 ft. high to assure a slope to and from each of the five relay pumping stations so that the concentrate lines will be self-draining in case of power difficulties. All told, 3 million board feet of lumber and 40 miles of wooden pipe were used.

When milling 10,000 tons of nickel-copper ore a day at Creighton approximately 1800 tons of concentrate and 8200 tons of tailing are produced. Water added to the concentrate makes a pulp which flows through the pipeline at the rate of 800 gallons per minute. The tailing pulp flows at the rate of 2500 gallons per minute.

Despite the rigorous northern winter weather, the complete pumping system has been designed for trouble-free operation. Several times the temperature has dropped to 30 degrees below zero with no operating difficulties.

Mica Mat for Insulation.—Mica mat, a new, paper-like material made of matted flakes of mica, is being used for ground insulation in the General Electric Company's Locomotive and Car Equipment Department, Erie, Pa.

Developed in Europe, mica mat is manufactured by G.E. in a newly-adapted process similar to paper making. General Electric engineers point out that the operation at Erie is one of the first American production uses of this type of ground insulation.

Mica mat is being used in both armature windings and field coils in the manufacture of some motors and generators for electric and diesel-electric locomotives at Erie.

The new material will stand about 600 volts per mil of thickness, G.E. engineers say. Its thickness is uniform within a fraction of a mil, and the thickness of a taped or wrapped conductor is now more constant than was possible with mica-glass cloth insulation. Engineers point out that mica mat also has fewer electrical "holes" than mica-glass cloth.

The manufacture of the new material, G.E. engineers explain, is done by baking mica at 700–800°C. and then suddenly quenching it with cold water, causing the mica to explode into tiny flakes. The water-mica slurry is then fed into a paper-making machine and comes out as a dry, fragile paper. This paper is impregnated with a heat-resisting silicone varnish and applied to glass cloth for use as wrappers. Engineers add that the finished product meets the specifications of class "H" insulation.

Fuse-Din Precision Thermometer.—To satisfy the demand for a precision thermometer which is not as brittle as conventional platinum and glass thermometers, an Armored Precision Thermometer has been designed by Weiller Instruments Corporation, New York City.

The over-all length of this Fuse-Din Armored Precision Thermometer is 14¼ in. It is protected by a stainless-steel tube ¼ in. o.d. The thermometer head is 1 in. in diameter and 1¼ in. long. The four terminals are fused into a disc of borosilicate glass.

Several types of Fuse-Din Precision Thermometers are available in this form. For temperatures up to 300°C., Type PBM, cast in lead thermometer, is recommended. The resistance unit is tungsten, fused into pryex.

For higher temperatures up to 400°C., the same resistance thermometer is available, cast in zinc.

Internal leads may be silver or gold, depending on the user's requirements.

To permit precision measurements up to a temperature of 1500°C., Type PPC is used. This is a platinum thermometer, fused into a ceramic matrix. The internal leads are platinum.

A special escape capsule for supersonic planes has been developed by the Navy Bureau of Aeronautics, has been successfully tested in wind-tunnel speeds of over 1200 mph. The unit is pressurized, contains oxygen equipment and a para-raft, can be used in fighter or multi-place planes. The capsule is constructed of armor plating and laminated fiberglass, thus protecting against fire and flak, will float, and can be fitted with heating unit for use in frigid areas. The parachute, in a reefed condition, acts as a stabilizer during the drop until a barometric control lets it open to full diameter to cushion the landing. Navy stresses fact that this capsule is not in competition with ejection cockpit capsule formerly announced, which is for persons in the cockpit only.